# LOYOLA COLLEGE (AUTONOMOUS), CHENNAI – 600 034

**M.Sc.** DEGREE EXAMINATION – **MATHEMATICS**

FIRST SEMESTER – **NOVEMBER 2023**

**PMT1MC01 – LINEAR ALGEBRA**

Date: 31-10-2023 Dept. No. Max. : 100 Marks

Time: 01:00 PM - 04:00 PM

| | |
|---|---|
| | **SECTION A – K1 (CO1)** |
| | **Answer ALL the questions** **(5 x 1 = 5)** |
| 1 | **Answer the following.** |
| a) | What is the characteristic polynomial of $\begin{bmatrix} 2 & 3 & 7 \\ 0 & 4 & 8 \\ 0 & 0 & -1 \end{bmatrix}$. |
| b) | Write the minimal polynomial of $\begin{bmatrix} 1 & 1 \\ 0 & 2 \end{bmatrix}$. |
| c) | Give an example for a T admissible subspace. |
| d) | Define cyclic vector. |
| e) | Find an Inner product on $R^3$. |
| | **SECTION A – K2 (CO1)** |
| | **Answer ALL the questions** **(5 x 1 = 5)** |
| 2 | **Choose the correct answer.** |
| a) | Let A be a real matrix of order 3 such that $A^2 = A$. Then possible eigen values of A are<br>i) 0,1 ii) 2, 1 iii) 3, 1, -1 iv) 1, 2, 3 |
| b) | Roots of a minimal polynomial of a matrix A are<br>i) nonzero ii) integers<br>iii) characteristic values iv) characteristic vectors |
| c) | If an operator $T$ on a vector space $V$ is diagonalizable if there exists a basis of V containing<br>i) eigen values of $T$ ii) eigen vectors of $T$<br>iii) unit vectors iv) normal vectors |
| d) | Let T be a linear operator on a finite dimensional vector space V and $\alpha \in V$ be a characteristic vector of T. Then dimension of $Z(\alpha\,; T)$ is<br>i) 1 ii) 2 iii) 0 iv) 0 or 1 |
| e) | In $R^2$ $(\alpha\|\beta) = ax_1y_1 + bx_2y_2$ where $\alpha = (x_1, x_2), \beta = (y_1, y_2)$ is an inner product if<br>i) $a = 2, b = -8$ ii) $a = 0, b = 3$ iii) $a = -7, b = 4$ iv) $a = 10, b = 3$ |
| | **SECTION B – K3 (CO2)** |
| | **Answer any THREE of the following** **(3 x 10 = 30)** |
| 3 | Let T be a linear operator on a finite-dimensional space V and let c be a scalar. Then prove that that following are equivalent.<br>(i) c is a characteristic value of T.<br>(ii)The operator (T - cI) is singular.<br>(iii) det (T - cI) = 0 |
| 4 | Write about the matrix of a projection operator. |
| 5 | Write any four properties of nilpotent operators |
| 6 | Let $T$ be a linear operator on $R^2$ defined by $T(x, y) = (0, x)$. Find a cyclic vector and a non cyclic vector of $T$ |

| | |
|---|---|
| 7 | State and prove any four properties of an adjoint operator. |
| | **SECTION C – K4 (CO3)** |
| | **Answer any TWO of the following** **(2 x 12.5 = 25)** |
| 8 | State and prove primary decomposition theorem |
| 9 | Let $V$ be a finite-dimensional vector space over the field $F$ and let $T$ be a linear operator on $V$. Then show that $T$ is triangulable if and only if the minimal polynomial for $T$ is a product of linear polynomials over $F$ |
| 10 | If $V = W_1 \oplus \ldots \oplus W_k$, then prove that there exist $k$ linear operators $E_1, \ldots, E_k$ on $V$ such that<br>i) each $E_i$ is a projection **.**<br>ii) $E_iE_j = 0$, if i ≠ j;<br>iii )$I = E_1 + \ldots + E_k$**;**<br>iv) the range of $E_i$ is $W_i$. Also discuss about the converse. |
| 11 | Let $V$ be a finite-dimensional vector space over the field $F$ and let $T$ be a linear operator on $V$ and $\alpha \in V$. Discuss about the relation between $T -$ annihilator of $\alpha$ and $Z(\alpha\,; T)$ |
| | **SECTION D – K5 (CO4)** |
| | **Answer any ONE of the following** **(1 x 15 = 15)** |
| 12 | Diagonalize the matrix $A = \begin{bmatrix} 5 & -6 & -6 \\ -1 & 4 & 2 \\ 3 & -6 & -4 \end{bmatrix}$. Also, write the minimal polynomial for $A$ |
| 13 | Let T be a linear operator on the finite-dimensional vector space V over the field F. Suppose that the minimal polynomial for T decomposes over F into a product of linear polynomials. Then prove that there is a diagonalizable operator D on V and a nilpotent operator N on V such that<br>(i) T = D + N ;<br>(ii) DN=ND<br>Also prove that the diagonalizable operator D and the nilpotent operator N are uniquely determined by (i) and (ii) and each of them is a polynomial in T. |
| | **SECTION E – K6 (CO5)** |
| | **Answer any ONE of the following** **(1 x 20 = 20)** |
| 14 | Discuss about cyclic decomposition theorem. |
| 15 | Create and analyse relations between nilpotent operator, rational form and Jordon canonical forms. |

#########