# LOYOLA COLLEGE (AUTONOMOUS), CHENNAI – 600 034

**M.Sc.** DEGREE EXAMINATION – **MATHEMATICS**

THIRD SEMESTER – **NOVEMBER 2023**

**PMT3MC01 – TOPOLOGY**

Date: 30-10-2023 Dept. No. Max. : 100 Marks

Time: 01:00 PM - 04:00 PM

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| | **SECTION A – K1 (CO1)** |
| | **Answer ALL the questions** **(5 x 1 = 5)** |
| 1 | **Answer the following** |
| a) | Define a topological space with an example. |
| b) | What do you mean by a subspace topology? |
| c) | Explain briefly a closed subset in a topological space. |
| d) | When can you say that a topological space is separated? |
| e) | Define a compact space. |
| | **SECTION A – K2 (CO1)** |
| | **Answer ALL the questions** **(5 x 1 = 5)** |
| 2 | **Choose the correct answer** |
| a) | (i) [1,2] is open in R<br>(ii) [1,2) is open in R<br>(iii) (1,2] is open in R<br>(iv) (1,2) is open in R |
| b) | *Y*= [-1, 1], a subspace of *R*<br>(i) $A = \left\{x / \frac{1}{2} < \lvert x\rvert < 1\right\}$ is open in Y & R<br>(ii) $B = \left\{x / \frac{1}{2} < \lvert x\rvert \leq 1\right\}$ is not open in Y<br>(iii) $C = \left\{x / \frac{1}{2} \leq \lvert x\rvert < 1\right\}$ is open in Y & R<br>(iv) $D = \left\{x / \frac{1}{2} \leq \lvert x\rvert \leq 1\right\}$ is open in Y & R |
| c) | (i) ∅ and *X* are open<br>(ii) ∅ and *X* are closed<br>(iii) arbitrary intersections open sets in *X* are open in *X*<br>(iv) arbitrary unions of open sets in *X* are open in *X* |
| d) | (i) (0,1]is compact<br>(ii) (0,1]is not compact<br>(iii) (0,1]is closed<br>(iv) (0,1]is not bounded |
| e) | (i)Any compact subset of a topological space is closed<br>(ii) Every closed subset of a compact space need not be compact<br>(iii) Every limit point compact space is compact space<br>(iv) Every limit point compact space is sequentially compact |
| | **SECTION B – K3 (CO2)** |
| | **Answer any THREE of the following** **(3 x 10 = 30)** |
| 3 | Prove the pasting lemma |

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| 4 | List the limit points of the following sets with explanation.<br>a) $A = (0,1]$ b) $B = \left\{\frac{1}{n}, n \in N\right\}$ c) $C = \{0\} \cup (1,2)$ d) $Q$ = the set of all rational numbers |
| 5 | If $f: X \to Y$ is a function where $X$ is metrizable, prove that f is continuous if and only if for every convergent sequence $x_n \to x$ in $X$, the sequence $f(x_n) \to f(x)$. |
| 6 | Demonstrate intermediate value theorem. |
| 7 | Is every closed interval in R uncountable? Justify it. |
| | **SECTION C – K4 (CO3)** |
| | **Answer any TWO of the following** **(2 x 12.5 = 25)** |
| 8 | How would you say that any interval or a ray in a linear continuum *L* in the order topology is connected? |
| 9 | Examine whether the following statement is true:<br>"If $f: X \to Y$ is a bijective continuous function, $X$ is compact and $Y$ is Hausdorff, then $f$ is a homeomorphism" |
| 10 | Is the product of finitely compact spaces compact? Justify it with a supportive proof |
| 11 | What are the separation axioms? Formulate them. |
| | **SECTION D – K5 (CO4)** |
| | **Answer any ONE of the following** **(1 x 15 = 15)** |
| 12 | Does there exist a metric $d'$ that induces a topology on a metric space $(X, d)$ such that every subset of X is bounded with respect to metric $d'$? Prove the supporting result |
| 13 | Examine whether the following statements are true?<br>(i) *X* is compact metrizable space implies *X* is limit point compact space.<br>(ii) *X* is sequentially compact metrizable space implies *X* is compact space |
| | **SECTION E – K6 (CO5)** |
| | **Answer any ONE of the following** **(1 x 20 = 20)** |
| 14 | Defend the topologies induced by the euclidean metric and the square metric are the same as the product topology on $R^n$. |
| 15 | Demonstrate Urysohn lemma. |

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