LOYOLA COLLEGE (AUTONOMOUS), CHENNAI – 600 034

B.Sc. DEGREE EXAMINATION – **MATHEMATICS**

FIFTH SEMESTER – APRIL 2010

MT 5406 / 5402 - COMBINATORICS

Date & Time: 29/04/2010 / 9:00 - 12:00 Dept. No. Max. : 100 Marks

SECTION A

Answer **ALL** the questions: **10 x 2 = 20**

1. Define Stirling number of second kind?
2. Give all the partitions for 4.
3. Define Falling factorial.
4. Define Exclusion Principle.
5. Find the number of increasing words of length 8 out of the set of alphabets {a, b, c, d} with $a < b < c < d$.
6. Define multinomial number.
7. How many words can be formed with the help of letters of the word MATHEMATICS?
8. In how many ways can we distribute n distinct objects into m distinct boxes with the objects in each box arranged in a definite order?
9. Evaluate $((720)$.
10. Find *per* $\begin{pmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{pmatrix}$.

SECTION B

Answer any **FIVE** questions: **5 x 8 = 40**

11. Prove that the cardinality of each of the following sets is $\frac{[m]^n}{n!}$.

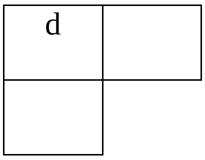
    (a) The set of increasing words of length n on m ordered letters.

    (b) The set of distributions of n non-distinct objects into m distinct boxes.

12. Give the recurrence formula for P_n^m and tabulate the values for n, m = 1, 2,…5.

13. (i) Define Euler's function and prove that $((n) = n\prod_{\downarrow}(i = 1)^{\uparrow}k \equiv (1 - 1/p_{\downarrow}i)$.
    (ii) Find the number of positive integers not greater than 100 which are not divisible by 2, 3, or 5. (5+3)

14. (i) Prove with usual notation that R(t, C) = t R(t, C_{dd}) + R(t, C'_d).
(ii) Find the Rook polynomial for the chessboard C given in the diagram below,



(4+4)

15. With proper illustration describe the problem of Fibonacci.
16. State and prove Multinomial theorem.
17. Describe the generating functions for partitions and derive the Bell's formula.
18. How many permutations of 1, 2, 3, 4 are there with 1 not in the 2nd position, 2 not in the 3rd position, 3 not in the 4th position and 4 not in the 4th position.

SECTION C

Answer any **TWO** questions: **2 x 20 = 40**

19. (a) Prove that the cardinality of each of the following sets is $\frac{[m]_n}{n!}$, the number is taken to be 0 if m$<$ n and 1 if n = 0.
(i) The set of n-subsets of an m-set.
(ii) The set of combinations of m symbols taken n at a time.
(b) How many ways can a total of 16 be obtained by rolling 4 dice. (12+8)

20. (i) Five gentlemen A, B, C, D, E attend a party, where before joining the party, they leave their overcoats in a checkroom. After the party, the overcoats get mixed up and are returned to the gentlemen in a random manner. What is the probability that none receives his own overcoat?
(ii) State and prove Generalized inclusion and exclusion principle. (10+10)

21. Define Menage problem and find the ménage number U_n .
22. (i) How many distinct circular necklace patterns are possible with 4 beads, these beads being available into 2 different colours, red and green.
(ii) Let G be a finite group and S a set. Let π be a homomorphism of G into the group of all permutations of S. Define $s_1 \approx s_2$ if and only if there exists a g $\in$ G such that $\pi_g s_1 = s_2$. Then prove that the number of $\approx$ equivalence classes is $\frac{1}{|G|}\sum_{g\in G}\Psi(g)$, where $\Psi(g)$ is the number of invariances in S for π_g .

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