



# LOYOLA COLLEGE (AUTONOMOUS), CHENNAI – 600 034

## B.Sc. DEGREE EXAMINATION – MATHEMATICS

SECOND SEMESTER – APRIL 2013

### MT 2502 - ALGEBRA AND CALCULUS - II

Date: 30/04/2013  
Time: 9:00 - 12:00

Dept. No.

Max. : 100 Marks

#### PART – A

(10 x 2 = 20 marks)

Answer ALL questions:

1. Evaluate  $\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \int_0^{2\cos\theta} r^2 d\theta dr.$

2. Define Beta function.

3. Prove that  $\int_0^{\frac{\pi}{2}} \sin^n x dx = \int_0^{\frac{\pi}{2}} \cos^n x dx.$

4. Change the order of integration and evaluate  $\int_0^1 \int_0^x dy dx.$

5. If  $f$  is an odd function show that  $\int_{-a}^a f(x) dx = 0.$

6. If  $n$  is a positive integer, show that  $\Gamma(n+1) = n!.$

7. State comparison tests for convergence and divergence of series.

8. If  $u_1 + u_2 + \dots$  is convergent, then prove that  $\lim_{n \rightarrow \infty} u_n = 0$

9. Write the expansion of  $(1-x)^n.$

10. Find the coefficient of  $x^n$  in the expansion of  $e^{a+bx}.$

#### PART – B

Answer any FIVE questions:

(5 x 8 = 40 marks)

11. Prove that  $\int_0^{\frac{\pi}{4}} \log(1 + \tan \theta) d\theta = \frac{\pi}{8} \log 2.$

12. Find the area of the cardioid  $r = a(1 + \cos \theta).$

13. Find the value of  $\iint xy dx dy$  taken over the positive quadrants of the ellipse  $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1.$

14. Evaluate  $\int_0^3 \int_{\sqrt{h-y}}^{\sqrt{h-y}} (x+y) dx dy,$  by changing the order of integration.

15. Prove that  $\beta(m, n) = \frac{\Gamma(m) \Gamma(n)}{\Gamma(m+n)}.$

16. Prove  $\int_0^{\frac{\pi}{2}} \sqrt{\sin \theta} d\theta \times \int_0^{\frac{\pi}{2}} \frac{d\theta}{\sqrt{\sin \theta}} = \pi.$

17. Test the convergence of the series  $\frac{1}{1.2} + \frac{1}{2.3} + \frac{1}{3.4} + \dots \infty$ .

18. Find the sum to infinity of the series  $1 + \frac{2}{6} + \frac{2.5}{6.12} + \frac{2.5.8}{6.12.18} + \dots \infty$ .

**PART – C**

**Answer any TWO questions:**

**(2 x 20 = 40 marks)**

19. a) Prove that  $\int \sin^n x = \frac{-\sin^{n-1} x \cos x}{n} + \frac{n-1}{n} I_{n-2}$ .

b) Prove that  $\int_0^{\pi} \frac{x \sin x}{1 + \cos^2 x} = \frac{\pi^2}{4}$ . (10+10)

20. a) Find the area common to  $y^2 = 4ax$  and  $x^2 = 4ay$ .

b) Evaluate  $\iiint \frac{dx dy dz}{(x+y+z+1)^3}$  over the region bounded by  $x = 0, y = 0, z = 0, x + y + z = 1$ . (10 +10)

21. a) Evaluate  $\int_0^1 x^m \left( \log \frac{1}{x} \right)^n dx$ .

b) Prove  $\sqrt{\frac{n+1}{2}} = \frac{(2n)! \sqrt{\pi}}{n! h^n}$  for  $n=0, 1, 2, \dots$  (10 +10)

22. a) Discuss the convergence of the series  $\frac{x}{2} + \frac{x^2}{3.4} + \frac{x^3}{5.6} + \dots \infty$ .

b) Show that  $\frac{1}{1.2.3} + \frac{1}{3.4.5} + \frac{1}{5.6.7} + \dots \infty = \log 2 - \frac{1}{2}$ . (10 +10)

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