LOYOLA COLLEGE (AUTONOMOUS), CHENNAI – 600 034

B.Sc. DEGREE EXAMINATION – **MATHEMATICS**

SECOND SEMESTER – **APRIL 2013**

MT 2503 - ANALY. GEOM. OF 3D, FOURIER SERIES & NUM. THEORY

Date: 03/05/2013 Dept. No. Max. : 100 Marks

Time: 9:00 - 12:00

SECTION – A

(Answer ALL questions) **(10 × 2 = 20)**

1. Write the equation of the plane in the intercept form.
2. State the equation of the straight line passing through the points (x_1,y_1,z_1) and(x_2 ,y_2 ,z_2).
3. Find the coordinates of the centre and radius of the sphere $2x^2+2y^2+2z^2-2x+4y+2z-15=0$.
4. Find the equation of the sphere with centre (-1,2,-3) and radius 3 units.
5. Define Fourier series expansion of a function f(x).
6. If f(x) is an even function what is the value of the Fourier coefficient b_n?
7. Find the sum of divisors of 360.
8. Find the number of integers less than n and prime to it when n=729 and 720.
9. If a,b,c are positive integers not all zero, show that (a+b+c)(ab+bc+ca) >9abc.
10. State Cauchy's inequality.

SECTION – B **(5X8=40)**

(Answer any FIVE questions)

11. Find the equation of the plane passing through the points (2,-5,-3), (-2,-3,5) and (5,3,-3).
12. Find the symmetric form of the equation of the line of intersection of the planes x+5y-z=7 and 2x-5y+3z+1=0.
13. Find the equation of the sphere with centre (6,-1,2) and touches the plane 2x-y+2z-2=0.
14. Find the equation of the sphere which passes through the circle $x^2+y^2+z^2-2x-4y=0$, x+2y+3z=8 and touches the plane 4x+3y=25.
15. Express f(x) =$\frac{1}{2}(\pi - x)$ as a Fourier series with period 2π to be valid in the interval 0 to 2π.

16. Prove that the sum of integers less than N and prime to it including unity is $\frac{1}{2}$ N ϕ(N).

17. Find the remainder obtained in dividing 2^{46} by 47.

18. If $a_1,a_2,.....a_n$ is an arithmetic progression, show that $a_1^2a_2^2....a_n^2 > a_1^n a_n^n$.Deduce that if n>2, $(n!)^2>n^n$.

SECTION – C

(Answer any TWO questions) (2× 20 = 40)

19. (a) Find the equation of the plane which passes through the point (-1,3,2) and perpendicular to the planes x+2y+2z =5 and 3x+3y+2z=8.

(b) Find the equation of the image of the line $\frac{x-1}{2} = \frac{y+2}{-5} = \frac{z-3}{2}$ in the plane 2x-3y+2z+3=0.

20 (a) Find the shortest distance between the lines $\frac{x-3}{-1} = \frac{y-4}{2} = \frac{z+2}{1}$ and $\frac{x-1}{1} = \frac{y+7}{3} = \frac{z+2}{2}$.

(b) A plane passes through a fixed point (a,b,c) and cuts the axes in A,B,C. Show that the locus of the centre of the sphere OABC is $\frac{a}{x}+\frac{b}{y}+\frac{c}{z}=2$.

21 (a) Show that $x^2 = \frac{\pi^2}{3} + 4\sum_{n=1}^{\infty}(-1)^n \frac{\cos nx}{n^2}$ in the interval [-π,π]. Deduce that

$\frac{1}{1^2} - \frac{1}{2^2} + \frac{1}{3^2} - ... = \frac{\pi^2}{12}$ and $\frac{1}{1^2} + \frac{1}{2^2} + \frac{1}{3^2} + ... = \frac{\pi^2}{6}$.

(b) Show that if x and y are both prime to the prime number n show that $x^{n-1} - y^{n-1}$ is divisible by n and deduce that $x^{12} - y^{12}$ is divisible by 1365.

22 (a) Show that 18! + 1 is divisible by 437,

(b) If x and y are positive quantities whose sum is 4, show that $\left(x+\frac{1}{x}\right)^2 + \left(y+\frac{1}{y}\right)^2 \geq 12\frac{1}{2}$.

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