LOYOLA COLLEGE (AUTONOMOUS), CHENNAI – 600 034

B.Sc. DEGREE EXAMINATION – **MATHEMATICS**

FIFTH SEMESTER – **APRIL 2013**

MT 5505/MT 5501 - REAL ANALYSIS

Date: 08/05/2013 Dept. No. Max. : 100 Marks

Time: 9:00 - 12:00

PART – A

Answer all questions **(10 × 2 = 20)**

1. Show that Z and N are similar.
2. Define subsequence of a sequence.
3. Define compact set and give an example for it.
4. Give an example of a (i) countably infinite set and (ii) uncountably infinite set.
5. Show that in a metric space, 'limit of a sequence is unique'.
6. Define complete metric space and give an example for it.
7. Give an example of a strictly increasing function and strictly decreasing function.
8. Show that every differentiable function is also continuous.
9. Define limit superior and limit inferior of a real sequence.
10. Define Riemann-Stieltjes integral of a function f with respect to α on [a, b].

PART – B

Answer any FIVE questions **(5x8 = 40)**

11. Define greatest common divisor of two integers and show that for any two integers a and b g.c.d.is of the form $\lambda a + \mu b$ where λ and μ are integers.
12. Show that $e = 1 + \frac{1}{1!} + \frac{1}{2!} + \ldots$is irrational.
13. Show that finite intersection of open sets is open and infinite intersection of open sets need not be open.
14. Show that every compact subset of a metric space is complete.
15. State and prove Minkowski's inequality.
16. Let f and g be functions of bounded variations defined on $[a, b]$. Show that $f + g$ and fg are also of bounded variations on $[a, b]$.
17. Show that $\sum_{k=1}^{n} \frac{1}{k} = \log n - \int_{1}^{n} \frac{x-[x]}{x^2} dx + 1.$

18. Suppose $c \in (a,b)$ and $\int_a^c f d\alpha, \int_c^b f d\alpha$ exist. Prove that $\int_a^b f d\alpha$ exists and

$$\int_a^c f d\alpha + \int_c^b f d\alpha = \int_a^b f d\alpha.$$

PART – C

Answer any TWO questions **(2x20=40)**

19. (A) State and prove Cauchy Schwarz inequality.

    (B) Prove that (0,1) is uncountable and hence deduce that R is uncountable.

20. (A) Show that a subset E of a metric space (X, d) is closed in X if and if it contain all its adherent points.

    (B) State and prove Bolzano- Wierstrass theorem.

21. (A) Show that Euclidean space R^k is complete.

    (B) Show that a continuous function defined on a compact metric space is uniformly continuous.

22. (A) State and prove Taylors theorem.

    (B) State Rolle 's Theorem.

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