



# LOYOLA COLLEGE (AUTONOMOUS), CHENNAI – 600 034

## B.Sc. DEGREE EXAMINATION – MATHEMATICS

FIRST SEMESTER – APRIL 2014

### MT 1100 - MATHEMATICS FOR PHYSICS

Date : 28/03/2014  
Time : 09:00-12:00

Dept. No.

Max. : 100 Marks

#### SECTION A

ANSWER ALL QUESTIONS.

(10 × 2 = 20)

1. Find the  $n^{\text{th}}$  derivative of  $e^{5x}$ .
2. Find the slope of the curve  $r = e^{\theta}$  at  $\theta = 0$ .
3. Write the expansion of  $(1 - x)^{-p/q}$ .
4. Find the rank the matrix  $\begin{pmatrix} 2 & 3 \\ 3 & 1 \end{pmatrix}$ .
5. Find  $L(r^2 + 2t)$ .
6. Find  $L^{-1}\left(\frac{1}{s(s+a)}\right)$ .
7. Write down the expansion of  $\tan 4\theta$ .
8. Prove that  $\cosh^2 x - \sinh^2 x = 1$ .
9. Two dice are thrown. What is the probability that the sum of the numbers is greater than 8?
10. If a Poisson variate  $X$  is such that  $P(X = 1) = 2P(X = 2)$ . Find the mean.

#### SECTION B

ANSWER ANY FOUR QUESTIONS.

(5 × 8 = 40)

11. Find the  $n^{\text{th}}$  differential coefficient of  $e^x \sin x \sin 2x$ .
12. Find the maximum value of  $\frac{\log x}{x}$  for positive values of  $x$ .
13. Prove that  $\log \frac{n+1}{n-1} = \frac{2n}{n^2+1} + \frac{1}{3} \left(\frac{2n}{n^2+1}\right)^3 + \frac{1}{5} \left(\frac{2n}{n^2+1}\right)^5 + \dots \infty$ .
14. Verify Cayley – Hamilton theorem for the matrix 
$$\begin{pmatrix} 1 & 0 & 3 \\ 2 & 1 & -1 \\ 1 & -1 & 1 \end{pmatrix}$$
15. If  $\cos(x + iy) = \cos \theta + i \sin \theta$ , prove that  $\cos 2x + \cosh 2y = 2$ .
16. Express  $\sin^7 \theta$  in a series of sines of multiples of  $\theta$ .
17. Find  $L^{-1}\left(\frac{s}{(s+2)^2}\right)$ .
18. Four cards are drawn at random from a pack of 52 cards. Find the probability that
  - (i) They are a king, a queen, a jack and an ace.
  - (ii) Two are kings and two are queens.
  - (iii) Two are black and two are red.
  - (iv) Two cards of hearts and two cards of diamonds.

#### SECTION C

ANSWER ANY TWO QUESTIONS.

(2 x 20 = 40)

19. (a) If  $y = \sin^{-1}x$ , prove that  $(1 - x^2)y_2 - xy_1 = 0$  and  $(1 - x^2)y_{n+2} - (2n + 1)xy_{n+1} - n^2y_n = 0$ .

(b) Find the angle of intersection of the cardioids  $r = a(1 + \cos\theta)$  and  $r = b(1 - \cos\theta)$ .  
(12 + 8)

20. (a) Find the eigen values and eigen vectors of the matrix  $A = \begin{pmatrix} 8 & -6 & 2 \\ -6 & 7 & -4 \\ 2 & -4 & 3 \end{pmatrix}$ .

(b) Separate  $\tan^{-1}(x + iy)$  into real and imaginary parts. (15 + 5)

21. (a) Express  $\cos 6\theta$  in terms of  $\sin\theta$ .

(b) Solve the equation  $\frac{d^2y}{dt^2} + 2\frac{dy}{dt} - 3y = \sin t$  given that  $y = \frac{dy}{dt} = 0$  when  $t = 0$ .  
(8 + 12)

22. (a) Find  $L^{-1}\left(\frac{1}{s(s+1)(s+2)}\right)$ .

(b) Calculate the mean and standard deviation for the following table giving the age distribution of 542 members:

Age (in years)	20 - 30	30 - 40	40 - 50	50 - 60	60 - 70	70 - 80	80 - 90
Number of Members	3	61	132	153	140	51	2

(6 + 14)

\*\*\*\*\*