



LOYOLA COLLEGE (AUTONOMOUS), CHENNAI – 600 034

B.Sc. DEGREE EXAMINATION – MATHEMATICS

SECOND SEMESTER – APRIL 2015

MT 2502 - ALGEBRA AND CALCULUS - II

Date : 15/04/2015
Time : 01:00-04:00

Dept. No.

Max. : 100 Marks

PART-A

Answer ALL questions:

(10 x 2 =20)

1. Evaluate $\int_0^{\pi/2} x(1-x^2)^{1/2} dx$.

2. Evaluate $\int \sec^6 x dx$.

3. Evaluate $\int_0^{2x} \int_{x^2} (2x+3y) dy dx$.

4. Find $\frac{\partial(u,v)}{\partial(x,y)}$ when $u = x^3 + y^2, v = y^3 + x^2$.

5. Prove that $\sqrt{n+1} = n\sqrt{n}$, if $n > 0$.

6. Evaluate $\int_0^{\pi/2} \sin^{10} \theta d\theta$ by using Gamma function.

7. Test the convergence of series $\sum_{n=0}^{\infty} \frac{n^3 + 1}{2^n + 1}$.

8. State the Raabe's Test for convergence and divergence of series.

9. Expand $(1-x)^{p/q}$ & $(1+x)^{-n}$.

10. Write the expansion for $\log\left(\frac{1+x}{1-x}\right)$.

PART-B

Answer any FIVE questions:

(5 x 8=40)

11. Find the area of cardioids $r = a(1 + \cos \theta)$.

12. Change the order of integration in the integral $\int_0^a \int_{x^2/a}^{2a-x} xy dx dy$ and evaluate it.

13. Evaluate $\iint_R xy dx dy$ Where R is the region in the first quadrant bounded by the hyperbolas $x^2 - y^2 = a^2$ and $x^2 - y^2 = b^2$ and the circles $x^2 + y^2 = c^2$ and $x^2 + y^2 = d^2$ ($0 < a < b < c < d$).

14. Express $\int_0^1 x^m (1-x^n)^p dx$ in terms of Gamma function and evaluate $\int x^5 (1-x^3)^{10} dx$.

15. Examine the convergence of the series $\sum \frac{(n+1)(n+2)\dots(n+n)}{n^n}$.

16. Discuss the convergence of the series $\frac{1}{1+x} + \frac{1}{1+2x^2} + \frac{1}{1+3x^3} + \dots$

17. Sum the series $1 + \frac{1+3}{2!} + \frac{1+3+3^2}{3!} + \dots \infty$.

18. Prove that $\frac{5}{1.2.3} + \frac{7}{3.4.5} + \frac{9}{5.6.7} + \dots \infty = 3 \log 2 - 1$.

PART-C

Answer any TWO questions:

(2 x 20=40)

19. (a) Evaluate $\int_0^{\frac{\pi}{2}} \log \sin x \, dx$.

(b) Prove that $\int_0^{\pi} \theta \sin^3 \theta \, d\theta = \frac{2\pi}{3}$. (10+10)

20. (a) By transforming into polar coordinates Evaluate $\iint \frac{x^2 y^2}{x^2 + y^2} \, dx \, dy$ over the annular

region between the circles $x^2 + y^2 = a^2$ & $x^2 + y^2 = b^2$, ($b > a$).

(b) Evaluate $\iint r \sqrt{a^2 - r^2} \, dr \, d\theta$ over the upper half of the circle $r = a \cos \theta$. (10+10)

21. (a) State and Prove the relation between Beta and Gamma function.

(b) Prove that $\beta\left(\frac{1}{2}\right) = \sqrt{\pi}$. (15+5)

22. (a) Find the sum to infinity of the series $1 + \frac{3}{4} + \frac{3.5}{4.8} + \frac{3.5.7}{4.8.12} + \dots$

(b) Show that $\log \sqrt{12} = 1 + \left(\frac{1}{2} + \frac{1}{3}\right) \frac{1}{4} + \left(\frac{1}{4} + \frac{1}{5}\right) \frac{1}{4^2} + \dots$ (10+10)

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