LOYOLA COLLEGE (AUTONOMOUS), CHENNAI – 600 034

M.Sc. DEGREE EXAMINATION – **MATHEMATICS**

SECOND SEMESTER – **APRIL 2015**

MT 2812 - PARTIAL DIFFERENTIAL EQUATIONS

Date : 21/04/2015 Dept. No. Max. : 100 Marks
Time : 01:00-04:00

ANSWER ALL THE QUESTIONS:

1. (a) Show that the equations $xp - yq = x$ and $x^2p + q = xz$ are compatible and find their solution. **(5)**

(or)

(b) Eliminate the arbitrary function f from the relation $z = xy + f(x^2 + y^2)$.

(c) Derive the condition for compatibility of two first order partial differential equations.

(or)

(d) Explain Jacobi's method of obtaining the solution of non – linear first order partial differential equations and hence solve $p^2x + q^2y = z$. **(15)**

2. (a) If $\alpha_r D + \beta_r D' + \gamma_r$ is a factor of $F(D, D')$ and $\phi_r(\xi)$ is an arbitrary function of the single variable ξ, then show that $u_r = \exp\left(\frac{-\gamma_r x}{\alpha_r}\right)\phi_r(\beta_r x - \alpha_r y)$ for $\alpha_r \neq 0$ is a solution of the equation $F(D, D')z = 0$.

(or)

(b) Show that $L(u) = c^2u_{xx} - u_{tt}$ is a self adjoint operator. **(5)**

(c) Obtain the canonical forms of hyperbolic, parabolic and elliptic partial differential equations.

(or)

(d) Solve $\frac{\partial^3 z}{\partial x^3} - 2\frac{\partial^3 z}{\partial x^2 \partial y} - \frac{\partial^3 z}{\partial x \partial y^2} + 2\frac{\partial^3 z}{\partial y^3} = e^{x+y}$. **(15)**

3. (a) Describe the three types of boundary value problem for Laplace equation.

(or)

(b) Derive telephone, telegraph and radio equations in transmission line problems. **(5)**

(c) A transmission line 1000 km long is initially under steady state conditions with potential 1300 volts at the sending end (x = 0) and 1200 volts at the receiving end (x = 1000). The terminal end of the line is suddenly grounded but the potential at source is kept at 1300 volts. Assuming the inductance and leakance to be negligible, find the potential v(x,t).

(or)

(d) Derive one dimensional wave equation. **(15)**

4. (a) Let $f(z)$ be analytic for $Re(z) \geq \gamma$, where γ is real constant greater than zero. Then for $(z_0) \geq \gamma$, prove that $f(z_0) = \frac{1}{2\pi i} \lim_{\beta\to\alpha} \int_{\gamma-i\beta}^{\gamma+i\beta} \frac{f(z)}{z-z_0} dz$ and the inverse Laplace transform is $f(t) = \frac{1}{2\pi i} \int_{\gamma-i\infty}^{\gamma+i\infty} \bar{f}(s)\, e^{st} ds$.

(or)

(b) Define piecewise continuous function, Laplace transform of an continuous function, inverse Laplace transform, wave equation and heat equation. **(5)**

(c) Use Laplace transform method to solve the initial value problem $k\frac{\partial u}{\partial t} = \frac{\partial^2 u}{\partial t^2}$, $0 < x < 1$, $0 < t < \infty$, subject to the conditions $u(0,t) = 0$, $u(l,t) = g(t)$, $0 < t < \infty$, and $u(x,0) = 0, 0 < x < l$. **(15)**

(or)

(d) (i) A string is stretched and fixed between two points (0,0) and$(l, 0)$. Motion is initiated by displacing the string in the form $u = \sin\left(\frac{\pi x}{l}\right)$ and released from rest at time $t = 0$. Find the displacement of any point on the string at any time t.

(ii) Prove that the application of an integral transform to a partial differential equation reduces the independent variables by one. **(10+5)**

5. (a) Find the iterated kernel of the following kernel $k(x,t) = \sin(x - 2t)$ if $0 \leq x \leq 2\pi$ and $0 \leq t \leq 2$.

(or)

(b) Prove that all iterated kernels of a symmetric kernel are also symmetric. **(5)**

(c) (i) Show that $y(x) = xe^{x^2}$ $y(x) = xe^{x^2}$ is a solution of Volterra integral equation $\int_0^x (1 - x^2 + t^2)\, y(t) dt = \frac{x^2}{2}$.

(ii) Find the resolvent kernel for Volterra integral equation with the following kernel $k(x,t) = \frac{\cosh x}{\cosh t}$.

(iii) Define the kernel of an integral equation. Mention the kernels of some of the integral transforms. **(6+5+4)**

(or)

(d) Find the solution of Volterra integral equation of the second kind by successive approximations. **(15)**

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