LOYOLA COLLEGE (AUTONOMOUS), CHENNAI – 600 034

B.Sc. DEGREE EXAMINATION – **MATHEMATICS**

FOURTH SEMESTER – **APRIL 2015**

MT 4503 - ALGEBRAIC STURUCTURE - I

Date : 16/04/2015 Dept. No. Max. : 100 Marks
Time : 09:00-12:00

PART – A

(Answer ALL questions) (10 × 2 = 20)

1. Define one to one and onto mapping.
2. Give an example of a finite group.
3. Show that any subgroup of an abelian group is normal.
4. Define a quotient group.
5. When do you say that two groups are isomorphic?
6. Express the permutation (1,3,4) (1,2,3,5) as product of disjoint cycles.
7. Give an example of an integral domain which is not a field.
8. Define an ideal of a ring.
9. When do you say that two elements of a commutative ring are associates?
10. What are Gaussian integers?

PART – B

(Answer any FIVE questions) (5 × 8 = 40)

11. If H is a non-empty finite subset of a group G and H is closed under the operation of G, show that H is a subgroup of G.
12. Show that union of two subgroups is a subgroup of G if and only if one is contained in the other.
13. Show that every group of prime order is cyclic.
14. If G is a group, show that set of all automorphisms on G, A(G) is also a group.
15. Show that every permutation can be expressed as product of disjoint cycles and this representation is unique up to the order of the factors.
16. Show that every finite integral domain is a field.
17. Let R be a commutative ring with unity and P is an ideal of R. Show that P is a prime ideal of R if and only if R/P is an integral domain.
18. Let R be a Euclidean ring. Show that any two elements a and b in R have a greatest common divisor d which can be expressed as $\lambda a + \mu b$ for some λ, μ in R.

PART – C

(Answer any TWO questions) **($2 \times 20 = 40$)**

19. (a) If G is a group in which $(ab)^k = a^k b^k$ for three consecutive integers k and for all a,b in G, show that G is abelian.

(b) If H and K are finite subgroups of a group G, show that $o(HK) = \frac{o(H)o(K)}{o(H \cap K)}$. (8+12)

20. (a) If N is a normal subgroup of a group G, show that G/N is also a group.

(b)State and prove the fundamental theorem of group homomorphism.

21. (a) Let G be a group. Show that the set of all inner automorphisms of G , I(G), is a normal subgroup of A(G) and $I(G) \approx G/Z(G)$ where Z(G) is the centre of G.

(b) Let R be a commutative ring with unity and M is an ideal of R. Show that M is a maximal ideal of R if and only if R/M is a field.

22. (a) Show that every Euclidean ring is a principal ideal domain.

(b) Show that Z(i) is a Euclidean ring. (8+12)

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