MT 6606 – COMPLEX ANALYSIS

PART – A

Answer ALL questions: (10 x 2 = 20 marks)

1. Show that the function $f(z) = \frac{\bar{z}}{z}$ does not have a limit as z $\to$0.
2. Test whether the following function $f(z) = e^x(\cos y + i\sin y)$ is analytic or not.
3. Find the critical points of $w = z + \frac{1}{z}$.
4. Define Cross ratio.
5. Define a simply connected region.
6. Using Cauchy's integral formula evaluate $\int_c \frac{z^2+5}{z-3}dz$ where c is $|z| = 4$.
7. Expand f(z) = $\frac{1}{z}$ in a Taylor's series about z = i.
8. Find all zero's of f(z) = cos z.
9. Find the residue of cot z at z = 0.
10. State fundamental theorem of algebra.

PART – B

Answer any FIVE questions: (5 x 8 = 40 marks)

11. Derive the Cauchy – Riemann equations in Polar form.
12. Show that $u = \log\sqrt{x^2+y^2}$ is harmonic and determine its harmonic Conjugate.
13. Find the image of the strip $2 < x < 3$ under $w = \frac{1}{z}$.
14. State and Prove Morera's theorem.
15. Using Cauchy's integral formula evaluate $\int_c \frac{e^z d^z}{(z+2)(z+1)^2}$, where C is $|z| = 3$.
16. If $f(z) = \frac{z+4}{(z+3)(z-1)^2}$ find Laurent's series expansions in (i) $0 < |z-1| < 4$ and (ii) $|z-1| > 4$.
17. State and prove Argument theorem.
18. Using contour integration prove that $\int_0^\infty \frac{\cos x}{1+x^2}dx = \frac{\pi}{2e}$.

PART – C

Answer any TWO questions: (2 x 20 = 40 marks)

19. State and prove the necessary and sufficient conditions for f(z) to be analytic.

20. (a) Find the bilinear transformation which maps the points z = −1, 1, ∞ respectively on w = –i, –1, i.

(b) Find an analytic function $f(z) = u + iv$ if $u + v = \dfrac{\sin 2x}{\cosh 2y - \cos 2x}$. (8+12)

21. (a) Using Cauchy's integral formula evaluate $\int_C \dfrac{z^3 dz}{(2z+i)^3}$, where C is the unit circle.

(b) Prove that $\int_0^{\pi} \dfrac{a\,d\theta}{a^2 + \sin^2\theta} = \dfrac{\pi}{\sqrt{a^2+1}}$ (a>0). (10+10)

22. (a) State and prove Laurent's series.

(b) Find the Laurent's series expansion of the function $\dfrac{z^2 - 1}{(z+2)(z+3)}$ valid in the annular region $2 < |z| < 3$. (12+8)

$$$$$$$$