LOYOLA COLLEGE (AUTONOMOUS), CHENNAI – 600 034

M.Sc. DEGREE EXAMINATION – **MATHEMATICS**

SECOND SEMESTER – **APRIL 2016**

MT 2962 – ACTUARIAL MATHEMATICS

Date: 27-04-2016 Dept. No. Max. : 100 Marks

Time: 01:00-04:00

Answer ALL Questions: **(5 x20=100 marks)**

1. (a) Briefly explain the history and origin of insurance.

(OR)

(b) Define and derive an expression for deferred probability. (5)

(c) If $S(x) = 1 - \frac{x^2}{100}$, $0 \le x \le 10$. Find the distribution of $K(4)$. Also obtain its expectation e_4.

(d) Prove that $\mu_x = \frac{f(x)}{s(x)}$ and $S(x) = \exp(-\int_0^x \mu_t dt)$. (8+7)

(OR)

(e) For the current type of refrigerator, it is given that $s(x) = \left\{ \begin{matrix} 1 & x \le 0 \\ 1 - \frac{x}{w} & 0 \le x \le w \\ 0 & x > w \end{matrix} \right\}$ and $e_0{}^0 = 20$.

For a proposed new type, with the same w , the new survival function is

$S^*(x) = \left\{ \begin{matrix} 1 & 0 \le x \le w \\ \frac{w-x}{w-5} & 5 < x \le w \end{matrix} \right\}$. Calculate the increase in life expectancy at time 0. (15)

2. (a) If $S(x) = 1 - \frac{x^2}{100}$, $0 \le x \le 100$ and l_0 = 1,00,000 , find l_1, l_4 and $l_{6.5}$.

(OR)

(b) Derive the relation l_x and μ_x. (5)

(c) Derive the expression for l_x, d_x , L_x , T_x , e_x and tabulate the values of l_x, d_x , L_x , T_x , e_x where $q_0 = 0.3$, $q_1 = 0.1$, $q_2 = 0.2$, $q_3 = 0.4$, $q_4 = 0.7$ *and* $q_5 = 1$ taking l_0 = 100 .

(d) Explain about assumption for fractional ages. (10+5)

(OR)

(e) Given that $p_{40} = 0.999473$, calculate ${}_{0.4}q_{40.2}$ under the assumption of under distribution of death.

(f) Given $q_{60} = 0.3$ and $q_{61} = 0.4$, find the probability that (60.5) will die between (60.5) and (61.5) under the assumption of uniformity of deaths in the unit interval. (7+8)

3. (a) Find the amount of Rs 10, 000/- after 10 years if the rate of interest is 5% and 5% per annum payable quarterly.

(OR)

(b) Find the amount to which Rs 1, 000/- will accumulate at 6% per annum convertible half yearly for 5 years. In how many will a sum of money double itself at compound interest with effective rate $= 0.005$? (5)

(c) Give an account of whole life insurance policy.

(d) Assume that each of 100 independent lives is of age x, is subject to a constant force of mortality $\mu = 0.04$ and is insured for a death benefit amount of 10 units, payable at the moment of death. The benefit payments are to be withdrawn from an investment fund earning interest at a rate $\delta = 0.06$. Calculate the minimum amount to be collected at t=0, so that the probability is approximately 0.95 that sufficient funds will be on hand to withdraw the benefit payment at the death of each individual. (5+10)

(OR)

(e) Derive the (i) effective rate of interest for both Simple and Compound Interest and (ii) discount in life insurance.

(f) Find the amount to which 1000 will accumulate at 6% per annum convertible half yearly for 5 years. (10+5)

4. (a) Find the present value and the accumulated value of a 10 year annuity immediate of $Rs.\,1000$ per annum if the effective rate of interest is 5%.

(OR)

(b) Rs. 3000 is deposited at a bank if January 1st of each year from 2001 – 2009. What is the accumulated value of this fund on December 31, 2009 at 3% annual rate of interest?

(5)

(c) For a 3-year temporary life annuity-due on (30), given $S(x) = 1 - \frac{x}{80}, 0 \le x < 80$ $i = 0.05$ and

$$Y = \begin{cases} \ddot{a}_{\overline{k+1}|}, k = 0,1,2 \\ \ddot{a}_{\overline{3}|}, \quad k=3,4,5 \end{cases}$$, calculate $Var(Y)$.

(d) Derive whole life annuity due. (10+5)

(OR)

(e) An alumni association has 50 members, each of age x. It is assumed that all lives are independent. It is decided to contribute Rs. R to establish a fund to pay a death benefit of rupees 10,000/- to each member. Benefits are to be payable at the moment of death. It is given that $\overline{A}_x = 0.06$ and ${}^2\overline{A}_x = 0.01$. Using normal approximation, find R so that with probability 0.95 the fund will be sufficient to pay the death benefit.

(f) Prove that $\ddot{a}_x = \frac{1 - A_x}{d}$. (8+7)

5. (a) For a whole life insurance with unit benefit, calculate $\bar{P}(\bar{A}_x)$ and $var\ (L)$ with the assumptions that the force of mortality is constant $\mu = 0.04$ and force of interest $\delta = 0.06$.

(OR)

(b) Calculate $\ddot{a}_x$ where it is given that ${}_{10}E_x = 0.40$, ${}_{\overline{10}|}\ddot{a}_x = 7$ and $\ddot{S}_{x:\overline{10}|} = 15$. (5)

(c) For (x) you are given the following information:

1) The premium for a $20-$year endowment insurance of 1 is 0.0349.
2) The premium for a $20-$year pure-endowment of 1 is 0.0230.
3) The premium for a $20-$year deferred whole life annuity-due of 1 per year is 0.2087 and is paid for 20 years.
4) All premiums are fully discrete annual benefit premiums.
5) $i = 0.05$.

Calculate the premium for a $20-$payment whole life insurance of 1.

(d) If ${}_{k|}q_x = c(0.96)^{k+1}, k = 0,1,2,\ldots$ where c=0.04/0.96 and i=0.06, calculate P_x and Var(L).

(7+8)

(OR)

(e) Given (i) ${}_{\overline{10}|}\ddot{a}_x = 4.0$, (ii) $\ddot{a}_x = 10.0$ (iii) $\ddot{S}_{x:\overline{10}|} = 15.0$, (iv) $\vartheta = 0.94$. Calculate $A'_{x:\overline{10}|}$.

(f) For a fully continuous whole life insurance 1 on (x). Calculate $\bar{P}(\bar{A}_x)$ given the following:

(i) Premiums are determine using the equivalence principle.
(ii) $\frac{var\ [Z]}{var[L]} = 0.36$ and
(iii) $\bar{a}_x = 10$. (5+10)