



Date: 06-05-2016

Dept. No.

Max. : 100 Marks

Time: 09:00-12:00

Section A

Answer ALL questions:

10 × 2 = 20

1. Find the n^{th} derivative of $\log(ax + b)$.
2. At which point of the curve $y = x^3 - 12x + 18$, is the tangent parallel to the x -axis.
3. Prove that $\frac{e-1}{e+1} = \frac{\frac{1}{1} + \frac{1}{3} + \frac{1}{5} + \dots}{\frac{1}{1} + \frac{1}{3} + \frac{1}{5} + \dots}$.
4. Define a skew-symmetric matrix and give an example.
5. Find the Laplace transform of $(t^2 + 2t + 3)$.
6. Find $L^{-1}\left(\frac{1}{(s-5)^2}\right)$.
7. Write down the expansion of $\sin \theta$ and $\cos \theta$ in a series of ascending powers of θ .
8. Prove that $\cosh^2 x - \sinh^2 x = 1$.
9. What is the chance that a leap year selected at random will contain 53 Sundays?
10. Determine the binomial distribution for which the mean is 4 and variance is 3.

Section B

Answer any FIVE questions:

5 × 8 = 40

11. Find the n^{th} differential coefficient of $\cos x, \cos 2x, \cos 3x$.
12. Find the angle of intersection for the curve $x^2 = 4y$ and $y^2 = 4x$.
13. Find the sum to infinity of the series $1 + \frac{2^2}{2!} + \frac{3^2}{3!} + \dots$.
14. Find the characteristic equation of the matrix $A = \begin{pmatrix} -2 & 2 & -3 \\ 2 & 1 & -6 \\ -1 & -2 & 0 \end{pmatrix}$ and verify that it is satisfied by A .
15. Find the Laplace transform of $f(t) = \begin{cases} e^{-t} & 0 < t < 4 \\ 0 & t > 4 \end{cases}$.
16. Express $\cos 3\theta$ in terms of $\sin \theta$.
17. If $\cos(x + iy) = \cos x + i \sin \theta$, prove that $\cos 2x + \cosh 2y = 2$.
18. Calculate the mean and standard deviation for the following table giving the age distribution of 542 members:

Age(in years)	20 – 30	30 – 40	40 – 50	50 – 60	60 – 70	70 – 80	80 – 90
No. of members	3	61	132	153	140	51	2

Section C

Answer any TWO questions:

$2 \times 20 = 40$

19. a) Find the maxima and minima of the function $x^3y^2(6-x-y)$.

b) Find the sum to infinity of the series $\frac{2-4}{3 \cdot 6} + \frac{2-4 \cdot 6}{3 \cdot 6 \cdot 9} + \frac{2-4 \cdot 6 \cdot 12}{3 \cdot 6 \cdot 9 \cdot 12} + \dots \infty$. (12 + 8)

20. a) Find all the eigen values and the associated eigen vectors of the matrix

$$A = \begin{pmatrix} 8 & -6 & 2 \\ -6 & 7 & -4 \\ 2 & -4 & 3 \end{pmatrix}$$

b) Expand $\sin^4\theta \cos^2\theta$ in a series of cosines of multiples of θ . (12 + 8)

21. a) Find $L^{-1}\left(\frac{1}{(s+1)(s^2+2s+2)}\right)$.

b) Solve the equation $\frac{d^2y}{dt^2} + 2\frac{dy}{dt} - 3y = \sin t$ given that $y = \frac{dy}{dt} = 0$ when $t = 0$. (10+ 10)

22. a) Separate into real and imaginary parts of $\tan^{-1}(x + iy)$.

b) A car hire firm has two cars, which it hires out day by day. The number of demands for a car on each day is distributed as a Poisson distribution with mean 1.5. Calculate the proportion of days on which
(i) neither car is used, and (ii) the proportion of days on which some demand is refused.

(10 + 10)
