LOYOLA COLLEGE (AUTONOMOUS), CHENNAI – 600 034

B.Sc. DEGREE EXAMINATION – **PHYSICS**

FOURTH SEMESTER – **APRIL 2016**

MT 4203 - ADVANCED MATHEMATICS FOR PHYSICS

(From 06th – 09th Batches)

Date: 27-04-2016 Dept. No. Max. : 100 Marks
Time: 09:00-12:00

SECTION – A

ANSWER ALL QUESTIONS: **(10 x 2 = 20)**

1. Evaluate $\int \log x \, dx$.
2. Define Fourier series.
3. State the necessary and sufficient condition for the ordinary differential equation to be exact.
4. Write the general solution when the roots are real and unequal.
5. Prove that $\beta(m.n) = \beta(n,m)$.
6. Define Gamma function.
7. Prove that the vector $\vec{F} = z\vec{i} + x\vec{j} + y\vec{k}$ is Solenoidal.
8. Show that the vector $2xy\vec{i} + (x^2 + 2yz)\vec{j} + (y^2 + 1)\vec{k}$ is irrotational.
9. Define cyclic group.
10. Give an example to show that every group need not be an abelian group.

SECTION – B

ANSWER ANY FIVE QUESTIONS: **(5 x 8 = 40)**

11. State and prove any two properties of definite integrals.
12. Evaluate □ $xy(x+y)dxdy$ over the area between the curves $y = x^2$ and $y = x$.
13. Find a sine series for $f(x) = x$ in the interval $(0, \pi)$.
14. Evaluate $\int x^2 e^{3x} dx$.
15. Solve $(D^2 + 3D + 2)y = x^2$.
16. If $\vec{F} = x^2y\vec{i} + y^2z\vec{j} + z^2x\vec{k}$, then find $curl\ curl\ \vec{F}$.
17. Express $\int_0^1 x^m(1 - x^n)^p \, dx$ in terms of the beta function and hence evaluate $\int_0^1 x^5(1 - x^3)^{10} \, dx$.
18. Prove that the set $\{1, \omega, \omega^2\}$ is an abelian multiplicative finite group of order 3.

SECTION – C

ANSWER ANY TWO QUESTIONS: **(2 x 20 = 40)**

19. (a) Find the Cosine series for $f(x) = \sin x$ in $0 < x < \pi$.

    (b) Define normal subgroup and left cosets. **(16+4)**

20. (a) Solve $(D^2 + 6D + 9)y = e^{2x} + \cos x$

    (b) Solve $\frac{dy}{dx} + y \cos x = \frac{1}{2} \sin 2x$. **(12+8)**

21. (a) Prove that $\beta(m, n) = \frac{\Gamma(m).\Gamma(n)}{\Gamma(n + n)}$.

    (b) Prove that $\Gamma\left(\frac{1}{2}\right) = \sqrt{\pi}$. **(15+5)**

22. Verify Gauss Divergence theorem for $\vec{F} = 4xz\vec{i} - y^2\vec{j} + yz\vec{k}$, over the cube bounded by $x = 0, x = 1, y = 0, y = 1, z = 0$ and $z = 1$. **(20)**

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