LOYOLA COLLEGE (AUTONOMOUS), CHENNAI – 600 034

B.Sc. DEGREE EXAMINATION – **MATHEMATICS**

FOURTH SEMESTER – **APRIL 2016**

MT 4503 – ALGEBRAIC STURUCTURE - I

Date: 20-04-2016 Dept. No. Max. : 100 Marks

Time: 09:00-12:00

PART – A

Answer ALL questions **(10 × 2 = 20 marks)**

1. Let Z be set of integers. Define a ≈ b if a – b is divisible by 5. Show that ≈ is an equivalence relation.
2. Prove that $(ab)^2 = a^2b^2$ for all a and b in a group G if and only if G is abelian.
3. Define order of an element of a group.
4. Show that every subgroup of an abelian group is normal.
5. Let Z be set of all integers and h :Z → Z be defined by h(x) = 2x.Show that it is a group homomorphism.
6. Define an automorphism of a group.
7. If R is ring, show that for a,b in R (i) (- a)b = a(- b) = - (ab) and (ii) (-a) (-b)=ab.
8. Give an example of an integral domain which is not a field.
9. Define prime ideal of a ring.
10. Define Euclidean ring.

PART – B

Answer any FIVE questions **(5 × 8 = 40 marks)**

11. If H and K are subgroups of a group G, show that HK is a subgroup of G if and only if HK = KH.
12. Show that the union of two subgroups of a group G is a subgroup of G if and only if one is contained in the other.
13. Show that every subgroup of a cyclic group is also cyclic.
14. Show a subgroup N of a group G is normal in G if and only if product of two left cosets of N in G is also a left coset of N in G.
15. Let h be a homomorphism of a group G onto a group G' . Let N' be a normal subgroup of G' and let N = {x ∈ G : h(x) ∈ N'}. Show that G/N ≈ G'/N'.
16. Let R be a commutative ring with unit element whose only ideals are (0) and R. Show that R is a field.
17. Show that an ideal of the ring Z of integers is a maximal ideal of Z if and only if it is generated by a prime number.
18. Let R be a Euclidean ring. Show that any two elements a and b in R have a greatest common divisor d = λa + μb for some λ and μ in R.

PART – C

Answer any TWO questions **(2 × 20 =40 marks)**

19. (a) Show that a nonempty subset H of a group G is a subgroup of G if and only if $a,b \in H$ implies that $ab^{-1} \in H$.

    (b) State and prove Lagrange's theorem. (8+12 marks)

20. (a) Let G be a group. Show that (i) the set of all inner automorphisms I(G) is a normal subgroup of A(G), the group of all automorphisms of G, (ii) $I(G) \approx G/Z(G)$, where Z(G) is the center of G.

    (b) Show that every permutation σ of a finite set E can be expressed as a product of disjoint cycles and this representation is unique upto the order of the factors. (10+10 marks)

21. (a) If A is an ideal of a ring R show that R/A is also a ring.

    (b) State and prove fundamental theorem of ring homomorphism. (10+10 marks)

22. (a) Let R be a commutative ring with unity and M be an ideal of R. Show that M is a maximal ideal of R if and only if R/M is a field.

    (b) Show that Z(i) is a Euclidean ring. (10+10 marks)

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