LOYOLA COLLEGE (AUTONOMOUS), CHENNAI – 600 034

B.Sc. DEGREE EXAMINATION – **MATHEMATICS**

FIFTH SEMESTER – **APRIL 2016**

MT 5505 – REAL ANALYSIS

Date: 26-04-2016 Dept. No. Max. : 100 Marks
Time: 09:00-12:00

SECTION – A

Answer ALL questions **(10 × 2 = 20 marks)**

1. Show that the sets Z and N are similar.
2. Differentiate between countable and uncountable sets.
3. Define discrete metric space.
4. Give an example of a compact set in the real space R.
5. Show that every convergent sequence is a Cauchy sequence in R.
6. Give an example of a continuous function which is not uniformly continuous.
7. When do you say that a function f :[a,b] → R has a right hand derivative at c in [a,b]?
8. When is a sequence $\{a_n\}$ said to be monotonic increasing or decreasing?
9. Define limit superior and limit inferior of a sequence.
10. Give an example of a function which is not Riemann integrable.

SECTION – B

Answer any FIVE questions. **(5 × 8 = 40 marks)**

11. If n ε N and n is not the square of any integer, show that $\sqrt{n}$ is irrational.
12. Show that the set R is uncountable.
13. Show that a subset E of a metric space (X,d) is closed in X if and only if it contains all its adherent points.
14. Show that composition of continuous functions is continuous.
15. If f and g are continuous functions at x_0 show that f+g and fg are also continuous at x_0.
16. State and prove Rolle's Theorem.
17. Show that a function f of bounded variation on [a,b] is bounded on [a,b].
18. Suppose $f \in R(\alpha)$ on [*a, b*]. Show that $\alpha \in R(f)$ on [*a, b*] and $\int_a^b f d\alpha + \int_a^b \alpha df = f(b)\alpha(b) - f(a)\alpha(a)$.

SECTION – C

Answer any TWO questions **($2 \times 20 = 40$ marks)**

19. (a) If $e = 1 + \frac{1}{1!} + \frac{1}{2!} + \frac{1}{3!} + \ldots$, show that e is irrational.

    (b) State and prove Cauchy- Schwarz inequality.

20. (a) If E is a subset of a metric space X, show that $\bar{E}$ is the smallest closed set containing E.

    (b) Show that the Euclidean space $\mathbf{R}^k$ is complete.

21. (a) Show that a map $f: X \to Y$ is continuous on X if and only if $f^{-1}(G)$ is open in X for every open set G in Y.

    (b) Let X be a compact metric space , Y be a metric space and $f: X \to Y$ be continuous on X . Show that f is uniformly continuous.

22. (a) State and prove Generalized Mean Value theorem and hence deduce Lagrange Mean Value theorem.

    (b) Suppose $f \in R(\alpha)$ on [a,b], α is differentiable on [a,b] and α' is continuous on [a,b]. Show that the Riemann integral $\int_a^b f\alpha'\, dx$ exists and $\int_a^b f\, d\alpha = \int_a^b f\alpha'\, dx$.

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