LOYOLA COLLEGE (AUTONOMOUS), CHENNAI – 600 034

M.Sc. DEGREE EXAMINATION – **MATHEMATICS**

SECONDSEMESTER – APRIL 2017

16PMT2MC03- PARTIAL DIFFERENTIAL EQUATIONS

Date: 21-04-2017 Dept. No. Max. : 100 Marks
01:00-04:00

Answer all questions. Each question carries 20 marks.

1. (a) Find the partial differential equation of the family of planes whose sum of x, y, z intercepts is equal to unity. (5)

(OR)

(b) Find the general integral of the linear PDE $(y+zx)p-(x+yz)q=x^2-y^2$. (5)

(c) Show that the following PDE's $xp-yq=x$ and $x^2p+q=xz$ are compatible and hence find their solution. (15)

(OR)

(d) (i) Find the characteristics of the equation $pq=z$ and hence determine the integral surface which passes through the parabola $x=0,\ y^2=z$. (10)

(ii) Find the complete integral of $(p^2+q^2)y=qz$ using Charpit's method. (5)

2. (a) Reduce the equation $x^2u_{xx}-2xyu_{xy}+y^2u_{yy}=e^x$ to a canonical form. (5)

(OR)

(b) Derive the canonical forms for second order PDE. (5)

(c) Explain the Riemann's method for solving $L(u)=F(x,y)$. (15)

(OR)

(d) Solve the equation $u_{xx}-2sinxu_{xy}-cos^2xu_{yy}-cosxu_y=0$. (15)

3. (a) Obtain the solutions of Laplace equation in cylindrical coordinates. (5)

(OR)

(b) Derive the Poisson equation $\nabla^2V=-4\pi G\rho$. (5)

(c) Find the solution for Neumann's problem for a rectangle. (15)

(OR)

(d) Solve the interior Dirichlet's problem for a circle. (15)

4. (a) Define boundary conditions and its various types. (5)

(OR)

(b) Derive the periodic solution of one-dimensional wave equation in spherical polar coordinates. (5)

(c) Find the solution of vibrating string problem under variables separable method. (15)

(OR)

(d) In an one- dimensional infinite solid, , $-\infty<x<\infty$, the surface $a<x<b$, is initially maintained at temperature T_0 and at zero temperature everywhere outside the surface. Show that $T(x,t)=\frac{T_0}{2}\left[erf\left(\frac{b-x}{\sqrt{4\alpha t}}\right)-erf\left(\frac{a-x}{\sqrt{4\alpha t}}\right)\right]$. (15)

5. (a) Explain briefly the eigen function method. (5)

(OR)

(b) Show that the Green's function has the symmetry property. (5)

(c) State and prove Helmholtz theorem. (15)

(OR)

(d) Solve the initial boundary value problem using the Laplace transform technique

$PDE;\ u_t = \alpha u_{xx}, \quad 0 < x < 1, t > 0$
$BCs; u(0,t) = 1, u(1,t) = 1,\ t > 0$ and IC; $u(x,0) = 1 + \sin\pi x, 0 < x < 1.$

(15)

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