



LOYOLA COLLEGE (AUTONOMOUS), CHENNAI – 600 034

B.Sc. DEGREE EXAMINATION – PHYSICS

THIRD SEMESTER – APRIL 2017

MT 3100- ALLIED MATHEMATICS FOR PHYSICS

Date: 03-05-2017
09:00-12:00

Dept. No.

Max. : 100 Marks

Part A

Answer **ALL** the questions

(10 x 2 = 20)

1. Evaluate $\lim_{x \rightarrow 0} \frac{\sin x - x}{x^3}$
2. Prove that $\frac{e-1}{e+1} = \frac{\frac{1}{2!} + \frac{1}{4!} + \frac{1}{6!} + \dots}{\frac{1}{1!} + \frac{1}{3!} + \frac{1}{5!} + \dots}$
3. Find $L[tsinat]$.
4. Prove that the sub tangent to the curve $y = a^x$ is of constant length.
5. Evaluate $L^{-1}\left[\frac{s}{(s+2)^2}\right]$.
6. If $y = \log(1+x)$ then find D^2y .
7. Expand $\tan 5\theta$ in terms of $\tan\theta$.
8. Prove that the matrix $\begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix}$ is orthogonal.
9. If $y = e^{2x}$, prove that $y_4 - 16y = 0$.
10. Write the general formula in Poisson's distribution.

Part B

Answer any **FIVE** questions

(5 x 8 = 40)

11. If $y = \sin^{-1} x$, prove that $(1-x^2)y_2 - xy_1 = 0$ and hence prove that $(1-x^2)y_{n+2} - (2n+1)xy_{n+1} - n^2y_n = 0$
12. Find $L[te^{-2t} \cos 3t]$.
13. If $\cos(x+iy) = \cos\theta + i\sin\theta$, then prove that $\cos 2x + \cosh 2y = 2$.
14. Sum the series $\frac{1.3}{2.4.6.8} + \frac{1.3.5}{2.4.6.8.10} + \frac{1.3.5.7}{2.4.6.8.10.12} + \dots \infty$
15. Verify Cayley Hamilton theorem for the matrix $A = \begin{pmatrix} 1 & -1 & 2 \\ -2 & 1 & 3 \\ 3 & 2 & -3 \end{pmatrix}$.
16. If $\sin(A+iB) = x+iy$, then
Prove that (i) $\frac{x^2}{\cosh^2 B} + \frac{y^2}{\sinh^2 B} = 1$ (ii) $\frac{x^2}{\sin^2 A} - \frac{y^2}{\cos^2 A} = 1$
17. Find $L^{-1}\left(\frac{s-3}{s^2+4s+13}\right)$
18. Ten coins are tossed simultaneously. Find the probability of getting at least seven heads.

Part C

Answer Any TWO Questions.

(2 x 20 = 40)

19. (a) Prove that $1 + \frac{2^4}{2!} + \frac{3^4}{3!} + \frac{4^4}{4!} + \dots \infty = 15e$

(b) Find the mean and standard deviation for the following table, giving the age distribution of 542 members.

Age in years	20-30	30-40	40-50	50-60	60-70	70-80	80-90
No. of members	3	61	132	153	140	51	2

(10+10)

20.(a) Prove that $\frac{\sin 7\theta}{\sin \theta} = 64\cos^6\theta - 80\cos^4\theta + 24\cos^2\theta - 1$

(b) Expand $\sin^3\theta \cos^4\theta$ in terms of sines of multiples of angles. (10+10)

21.(a) Find the maximum and minimum value of the function $2x^3 - 3x^2 - 36x + 10$.

(b) Find the angle of intersection of the cardioids $r = a(1 + \cos \theta)$ and $r = b(1 - \cos \theta)$ (10+10)

22.(a) Solve the equation $\frac{d^2y}{dt^2} + 2\frac{dy}{dt} - 3y = \sin t$, given that $y = \frac{dy}{dt} = 0$ when $t = 0$.

(b) Find $L^{-1}\left[\frac{1}{s(s+1)(s+2)}\right]$ (15+5)

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