LOYOLA COLLEGE (AUTONOMOUS), CHENNAI – 600 034

M.Sc. DEGREE EXAMINATION – **MATHEMATICS**

FOURTHSEMESTER – APRIL 2017

MT 4815- ADVANCED GRAPH THEORY

Date: 20-04-2017 Dept. No. Max. : 100 Marks
09:00-12:00

ANSWER ALL QUESTIONS

I a) Show that in any group of two or more people, there are always two with exactly the same number of friends inside the group

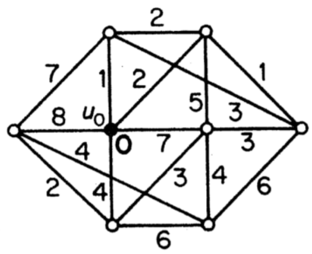
[OR]

b)Prove that every 3-regular graph has an even number of points. **(5)**

c) i) State and prove Chavatal theorem for hamiltonian graphs.
ii)Prove that any self complementary graph has 4n or 4n + 1 points. **(10+ 5)**

[OR]

d) (i) Write Dijkstra's algorithm.
(ii) Apply Dijkstra's algorithm to find shortest path from u to all other vertices of the following graph.



(5+ 10)

II a) Write Fleury's Algorithm.
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[OR]

b) Show that a connected graph has an Euler's trail if and only if it has atmost two vertices of odd degree . **(5)**

c) i) With usual notations prove that $\kappa \leq \kappa' \leq \delta$
ii)Prove that $\tau(K_n) = n^{n-2}$ **(7+8)**

[OR]

d) i)State and prove Dirac theorem for hamiltonian graphs.
ii)Show that $c(G)$ is well defined. **(7+8)**

III a) State and prove marriage theorem.

[OR]

b) If G is bipartite, then show that $\chi' = \Delta$. **(5)**

c) i) Prove that a matching M in G is a maximum matching iff G contains no M – augmenting path.
ii) Let M be a matching and K be a covering with |M|=|K|. Then show that M is a maximum matching and K is a minimum covering. **(10+5)**

[OR]

d) i)State and prove Tutte theorem. .

ii) Show that every 3 – regular graph without cut edges has a perfect matching.

(12 + 3)

IV a)With usual notations, show that $\alpha + \beta = \gamma$.

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[OR]

b) If G is k-critical then prove that $\delta \geq k - 1$ **(5)**

c)i)State and prove Dirac theorem for vertex coloring.

ii)Prove that every critical graph is a block. **(10 + 5)**

[OR]

d)i)Define type 1 and type 2 components and give examples.

ii)State and prove Brook's theorem. **(3 + 12)**

V a) Prove that K_5 is non planar.

[OR]

b) State and prove Euler's theorem for planar graphs **(5)**

c) i)State and prove five colour theorem.

ii)Define inner bridges and give an example. **(10 + 5)**

[OR]

d) State and prove Kuratowski's theorem . **(15)**

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