LOYOLA COLLEGE (AUTONOMOUS), CHENNAI – 600 034

B.Sc. DEGREE EXAMINATION – **MATHEMATICS**

FIFTHSEMESTER – APRIL 2017

MT 5406- COMBINATORICS

Date: 03-05-2017 Dept. No. Max. : 100 Marks
01:00-04:00

PART – A

Answer ALL questions. $(10 \times 2 = 20)$

1. What is called a combinatorial distribution? Give an example.
2. How many 4 – letter words with distinct letters can be got from the word "UNIVERSAL"?
3. Write the sequence of $(3+x)^3$.
4. Define Exponential generating function.
5. Find the coefficient of x^5 in $(1+x)(1+2x)(1+3x)(1+4x)(1+7x)$.
6. Identify the coefficient of $x_1^2 x_3 x_4^3 x_5^4$ in $(x_1+x_2+x_3+x_4+x_5)^{10}$.
7. Define De-arrangement.
8. Given $A = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$. Calculate permanent of A? .
9. What is the cyclic index of the symmetric group S_n?
10. Define circular words of length n.

PART – B

Answer any FIVE questions. $(5 \times 8 = 40)$

11. Formulate a table for S_n^m for $1 \le n \le 5$ using Sterling formula of first kind..
12. In a town council there are 10 democrats and 11 republicans. There can 4 women among democrats and 3 women among the republicans. Find the number of ways forming a planning committee of 8 members which has equal number of men and women and equal number from the both parties.
13. Derive Pascal's Identity using the concept of generating functions.
14. Obtain the Ordinary generating function (OGF) for the following sequences:
    (a) $(1, 1, 1, 1, \cdots\cdots)$
    (b) $(1, -1, 1, -1, \cdots\cdots)$
    (c) $(1, 2, 3, 4, \cdots\cdots)$
    (d) $(1, -2, 3, -4, \cdots\cdots)$
    (e) $(0, 1, 2, 3, 4, \cdots\cdots)$
15. In a university 60% of the students play Tennis(T), 50% play Cricket (C), 70% of the student play Basketball (B), 20% play both (T) and (C), 30% play both (T) and (B), 40% play (C) and (B). Find the percentage of the student play all the three games.

16. Find the rook polynomial for 2 X 2 chessboard by the use of expansion formula.

17. State and prove POLYA'S Enumeration theorem.

18. State and prove Burnside's lemma.

PART – C

Answer any TWO questions. $(2 \times 20 = 40)$

19. (a) Prove that the number of distributions of n distinct objects into m distinct boxes with the objects in the each box arranged in a definite order is $[m]^n$.

    (b) There are 5 mathematics students and 7 statistics students in a group. Find the number of ways of selecting 4 students from the group, if

    (i) there is no restriction

    (ii)all four must be mathematics students

    (iii) all four must be statistics students

    (iv) all four must belong to the same subject. (12+8)

20. (a) State and prove Multinomial theorem.

    (b) How many integers between 1 and 300 are

    (i) divisible by atleast one of 3,5,7.

    (ii) divisible by 3 and 5 but not by 7

    (iii) divisible by 5 but neither by 3 nor by 7? (12+8)

21. State and prove Ménage problem.

22. (a) Describe the problem of Fibonacci with an illustration.

    (b) Let G be symmetric group of a square with vertices labelled as $1, 2, 3 \& 4$ in clockwise. Find the elements of G and the cycle index of G. (8+12)

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