LOYOLA COLLEGE (AUTONOMOUS), CHENNAI – 600 034

B.Sc. DEGREE EXAMINATION – **MATHEMATICS**

FIFTHSEMESTER – APRIL 2017

MT 5505- REAL ANALYSIS

Date: 22-04-2017 Dept. No. Max. : 100 Marks
01:00-04:00

PART – A

ANSWER ALL THE QUESTIONS (10 x 2 = 20)

1. Write the triangle inequality.
2. Define countable and uncountable sets.
3. What is a discrete metric space.
4. Define Interior point.
5. Define Homeomorphism.
6. Give an example of a continuous function which is not uniformly continuous.
7. Define local minimum and local maximum of a function at a point.
8. State Rolle's theorem.
9. Define bounded variation.
10. State linearity property of Riemann – Stieltjes integral·

PART – B

ANSWER ANY FIVE QUESTIONS (5 x 8 = 40)

11. Show that e is irrational.
12. Prove that the countable union of countable sets is countable.
13. Let Y be a subspace of a metric space (X, d). Prove that a subset A of Y is open in Y iff $A = Y \cap G$ for some set G open in X.
14. Prove that the continuous image of a compact metric space is compact.
15. Let $f:(X,d_1)\to(Y,d_2)$ be uniformly continuous on X. If $\{x_n\}$ is a Cauchy sequence in X. Prove that $\{f(x_n)\}$ is a Cauchy sequence in Y.
16. State and prove Lagrange's mean value theorem .
17. If 'f' is monotonic on [a,b], then prove that the set of discontinuous functions of 'f' is countable.
18. State and prove the formula for Integration by parts.

PART – C

ANSWER ANY TWO QUESTIONS (2x 20 = 40)

19. (a) State and prove Cauchy – Schwarz Inequality.
    (b) Prove that **R** is uncountable.
20. (a) Prove that the Euclidean space $\Re^k$ is complete.

(b) Every compact subset of a metric space is complete.

21. State and prove Bolzano theorem and deduce Intermediate value theorem.
22. (a)State and prove Taylor's formula with remainder.

(b) Suppose (i) $f \in R(\alpha)$ on [a,b], (ii) α is differentiable on [a,b] and (iii) α^1 is continuous on [a, b]. Prove that the Riemann integral $\int_a^b f\alpha' dx$ exists and $\int_a^b f d\alpha = \int_a^b f\alpha' dx$.

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