LOYOLA COLLEGE (AUTONOMOUS), CHENNAI – 600 034

B.Sc.DEGREE EXAMINATION – **MATHEMATICS**

SIXTHSEMESTER – APRIL 2017

MT 6603 / MT 6600- COMPLEX ANALYSIS

Date: 18-04-2017 Dept. No. Max. : 100 Marks
09:00-12:00

PART-A

Answer all the questions (10x2=20)

1. Show that $u = x^3 - 3xy^2 + 3x^2 - 3y^2 + 1$ is harmonic.
2. Find the modulus of $\frac{(2-i)(1+i)}{1-i}$.
3. Using Cauchy's integral formula evaluate $\int_C \frac{z\,dz}{2z+1}$ where C is $|z| = 2$.
4. Prove that $\int_{-C} f(z)\,dz = -\int_C f(z)\,dz$.
5. Write the Maclaurin's series expansion of $f(z)$.
6. Define Meromorphic function.
7. Calculate the residue of $\frac{z+1}{z^2 - 2z}$ at its poles.
8. State Rouche's theorem.
9. Determine the angle of rotation and scale factor at the point $z = 2 + i$ under the mapping $w = z^2$.
10. Find the invariant points of the transformation $w = \frac{z}{2-z}$.

PART-B

Answer any FIVE questions (5 x 8=40)

11. If $f(z) = u(x, y) + iv(x, y)$ is an analytic function and $u(x, y) = e^y (x\cos y - y\sin y)$, find $f(z)$.
12. Derive the C-R equations in polar coordinates.
13. State and prove Cauchy's integral formula.
14. Expand $f(z) = \sin z$ in a Taylor's series about $z = \pi/4$ and determine the region of convergence of the series.
15. State and prove Maximum modulus theorem.
16. Find the residue of $\frac{1}{z - \sin z}$ at its poles.
17. Find the bilinear transformation which maps $-1, 0, 1$ of the z-plane onto $-1, -i, 1$ of the w-plane. Show that under this transformation the upper half of the w-plane maps onto the exterior of the unit circle $|z| = 1$.
18. Prove that any bilinear transformation can be expressed as a product of translation, rotation, magnification or contraction and inversion.

PART-C

Answer any TWO questions **(2 x 20=40)**

19. State and prove the necessary and sufficient condition for differentiability of a complex valued function.

20. (a) State and prove Cauchy's theorem.

(b) Show that $\int_C |z|^2 \, dz = -1 + i$ where C is the square with vertices $O(0,0)$, $A(1,0)$, $B(1,1)$, $C(0,1)$.

(12+8)

21. (a) State and prove Laurent's theorem.

(b) Find the Laurent's series expansion of $f(z) = z^2 e^{\frac{1}{z}}$ about $z=0$. **(15+5)**

22. (a) Prove that $\int_0^\infty \frac{x^4}{x^6 - 1} dx = \frac{\pi\sqrt{3}}{6}$.

(b) Prove that any bilinear transformation preservers cross ratio. **(15+5)**

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