LOYOLA COLLEGE (AUTONOMOUS), CHENNAI – 600 034

M.Sc.DEGREE EXAMINATION – **MATHEMATICS**

SECONDSEMESTER – APRIL 2018

17/16PMT2MC03- PARTIAL DIFFERENTIAL EQUATIONS

Date: 21-04-2018 Dept. No. Max. : 100 Marks

Time: 01:00-04:00

Answer ALL the questions:

1. (a) Find the partial differential equation of the family of planes, the sum of whose x, y, z intercepts is equal to unity. (5)

(OR)

(b) Use Lagrange's method to solve the equation $\begin{vmatrix} x & y & z \\ \alpha & \beta & \gamma \\ \frac{\partial z}{\partial x} & \frac{\partial z}{\partial y} & -1 \end{vmatrix} = 0$where $z = z(x, y)$. (5)

(c) Find the characteristics of the equation $pq = z$ and determine the integral surface which passes through the parabola $x = 0,\ y^2 = z$. (15)

(OR)

(d) Derive the necessary and sufficient conditions for the two partial differential equations $f(x, y, z, p, q) = 0$ and $g(x, y, z, p, q) = 0$ to be compatible. (15)

2. (a) Find the characteristic equations of $u_{xx} + 2\, u_{xy} + sin^2 x\, u_{yy} + u_y = 0$. (5)

(OR)

(b) Construct the adjoint operator for $L(u) = a(x)\frac{d^2u}{dx^2} + b(x)\frac{du}{dx} + c(x)u$. (5)

(c) Obtain the Riemann solution for the equation $\frac{\partial^2 u}{\partial x \partial y} = F(x, y)$ given that (i) $u = f(x)$on Γ, (ii) $\frac{\partial u}{\partial x} = g(x)$ on Γ, where Γ is the curve $y = x$. (15)

(OR)

(d) Explain the canonical form for second order elliptic partial differential equation and reduce the following equation $(1 + x^2)u_{xx} + (1 + y^2)u_{yy} + xu_x + yu_y = 0$ to a canonical form. (15)

3. (a) Derive Poisson's equation. (5)

(OR)

(b) Obtain the solutions of Laplace's equation in cylindrical coordinates. (5)

(c) Solve the interior Dirichlet's problem for a circle. (15)

(OR)

(d) Solve the PDE $\nabla^2 u = 0, 0 \le x \le a, 0 \le y \le b$, subject to the boundary condition $u_x(0, y) = u_x(a, y) = 0,\ u_y(x, 0) = 0,\ u_y(x, b) = f(x)$. (15)

4. (a) Solve the 3-dimensional diffusion equation $\frac{\partial T}{\partial t} = \alpha \nabla^2 T$ where $T = T(r, \theta, z, t)$. (5)

(OR)

(b) Derive the periodic solutions of one-dimensional wave equation in spherical polar coordinates. (5)

(c) Obtain the solution of the equation $u_{tt} - c^2 u_{xx} = 0$ under the following conditions: (i) $u(0, t) = 0$, (ii) $u(2, t) = 0$, (iii) $u(x, 0) = sin^3 \frac{\pi x}{2}$, and (iv) $u_t(x, 0) = 0$. (15)

(OR)

(d) Solve the one-dimensional diffusion equation in the region $0 \leq x \leq \pi, t \geq 0$ subject to the conditions (i) T remains finite as $t \to \infty$, (ii) $T = 0$, if $x = 0, \pi$ for all t, and (iii) At $t = 0$, $T = \begin{cases} x & : \quad 0 \leq x \leq \pi/2 \\ \pi - x & : \quad \pi/2 \leq x \leq \pi \end{cases}$. (15)

5. (a) Use Green's function technique to solve the Dirichlet's problem for a semi-infinite space. (5)

(OR)

(b) Find the Green's function for the Dirichlet problem on the rectangle $\mathbb{R}$: $0 \leq x \leq a,\ 0 \leq y \leq b$, described by the partial differential equation $(\nabla^2 + \lambda)u = 0$ in $\mathbb{R}$ and the boundary condition $u = 0$ on $\partial\mathbb{R}$. (5)

(c) Show that the solution of the Dirichlet's problem is reduced to the determination of Green's function. (15)

(OR)

(d) State and prove Helmholtz theorem. (15)

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