LOYOLA COLLEGE (AUTONOMOUS), CHENNAI – 600 034

B.Sc. DEGREE EXAMINATION – **MATHEMATICS**

THIRD SEMESTER – **APRIL 2018**

MT 3501– ALGEBRA, CALCULUS AND VECTOR ANALYSIS

Date: 05-05-2018 Dept. No. Max. : 100 Marks
Time: 01:00-04:00

PART – A

Answer all questions: **(10 X 2 = 20)**

1. Evaluate $\int_0^a \int_0^b xy(x-y)\,dy\,dx$.

2. Evaluate $\int_0^{\pi/2} \sin^7\theta \cos^5\theta \, d\theta$.

3. Solve $x + y\dfrac{\partial z}{\partial x} = 0$.

4. Solve $p = q^2$.

5. Prove that $\nabla \times \vec{r} = \vec{0}$ where $\vec{r} = x\hat{i} + y\hat{j} + z\hat{k}$.

6. Find the value of "a" if the vector $\vec{v} = 3x\vec{i} + (x+y)\vec{j} - az\vec{k}$ is solenoidal.

7. Find $L\left(t^3 - 3t^2 + 2\right)$.

8. Find $L\left(e^{-2t}\cos 3t\right)$.

9. Find the value of $\varphi(600)$.

10. State Fermat's theorem.

PART – B

Answer any five questions: **(5 X 8 = 40)**

11. Evaluate $\iiint xyz\,dx\,dy\,dz$ taken through the positive octant of the sphere $x^2 + y^2 + z^2 = a^2$.

12. Evaluate $\int_0^1 x^m \left(\log\dfrac{1}{x}\right)^n dx$.

13. Solve $p + q = px + qy$.

14.Solve $z = px + qy + \sqrt{1+p^2+q^2}$.

15.Compute the divergence and curl of the vector $\vec{F} = xyz\,\hat{i} + 3x^2y\,\hat{j} + \left(xz^2 - y^2z\right)\hat{k}$ at $(1,2,-1)$.

16.Prove that $\vec{F} = \left(y^2\cos x + z^3\right)\hat{i} + \left(2y\sin x - 4\right)\hat{j} + \left(3xz^2 + 2\right)\hat{k}$ is irrotational and find its scalar potential.

17.Find $L^{-1}\left(\dfrac{s}{(s+3)^2+4}\right)$.

18.Find the number of all divisors of 480. Also find the sum of all the divisors of 480.

PART – C

Answer any two questions: **(2 X 20 = 40)**

19.(i)By changing into polar co-ordinates, evaluate the integral $\int_0^{2a}\int_0^{\sqrt{2ax-x^2}} \left(x^2+y^2\right)dxdy$.

(ii) Express $\int_0^1 x^m\left(1-x^n\right)^p dx$ in terms of Gamma functions and evaluate the integral $\int_0^1 x^5\left(1-x^3\right)^{10} dx$. **(10+10)**

20.(i) Find the general solution of $xzp + yzq = xy$.

(ii) Solve $pxy + pq + qy = yz$. **(10+10)**

21.(i) Verify Gauss- Divergence theorem for the function $\vec{F} = 2xz\,\hat{i} + yz\,\hat{j} + z^2\,\hat{k}$ over the upper half of the sphere $x^2+y^2+z^2=a^2$.

(ii) Show that $n(n+1)(2n+1)$ is divisible by 6**.** **(15+5)**

22.(i) Solve the simultaneous equations $$3\frac{dx}{dt} + \frac{dy}{dt} + 2x = 1$$ $$\frac{dx}{dt} + 4\frac{dy}{dt} + 3y = 0$$

given $x = 0 = y$ at $t = 0$.

(ii) State and prove Wilson's theorem. **(10+10)**

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