LOYOLA COLLEGE (AUTONOMOUS), CHENNAI – 600 034

B.Sc. DEGREE EXAMINATION –**MATHEMATICS**

FIFTH SEMESTER – **APRIL 2018**

MT 5509– ALGEBRAIC STRUCTURE - II

Date: 03-05-2018 Dept. No. Max. : 100 Marks

Time: 09:00-12:00

PART – A

Answer ALL the questions: (10 x 2 = 20 marks)

1. Define a vector space over a field F.
2. Show that the vectors (1, 1) and (-3, 2) in $\Re^2$ are linearly independent over $\Re$, the field of real numbers.
3. Define a basis of a vector space.
4. Define a homomorphism of a vector space into itself.
5. Define an orthonormal set.
6. Normalize $\left(\frac{1}{2}, \frac{-1}{3}, \frac{1}{4}\right)$ in $\Re^3$ relative to the standard innear product.
7. Define an algebra over a field F.
8. Show that $A = \begin{pmatrix} \frac{1}{\sqrt{2}} & \frac{i}{\sqrt{2}} \\ \frac{i}{\sqrt{2}} & \frac{1}{\sqrt{2}} \end{pmatrix}$ is unitary.
9. Define a skew symmetric matrix and give an example.
10. Define Hermitian and skew Hermitian matrices.

PART – B

Answer any FIVE questions: (5 x 8 = 40 marks)

11. If S and T are subsets of a vector space V over F, then prove that L(SUT) = L(S) + L(T).
12. Prove that the union of two subspaces of a vector space V over F is a subspace of V if and only if one is contained in the other.

13. If V is a vector space of finite dimension and W is a subspace of V, then prove that dim

$$\frac{V}{W} = \dim V - \dim W.$$

14. Prove that $V \cong F^n, V, a$ vector space of dimension n and F^n is a vector space of ordered n – tuples.

15. Prove that T∈A(V) is invertible if and only if the constant term of the minimal polynomial for T is non-zero.

16. For A, B∈F_n , λ∈F, prove the following $(i)\, t_r(\lambda A) = \lambda\, t_r(A)$ $(ii)\, t_r(A+B) = t_r(A) + t_r(B)$.

17. Show that any square matrix can be expressed uniquely as the sum of symmetric and a skew symmetric matrices.

18. Show that the system of equations $x + 2y + z = 11,\ 4x + 6y + 5z = 8,\ 2x + 2y + 3z = 19$ is inconsistent.

PART – C

Answer any TWO questions: (2 x 20 = 40 marks)

19. a) Prove that a nonempty subset W of a vector space V over F is a subspace of V if and only if W is closed under addition and scalar multiplication. **(10)**

b) Prove that the vector space V over F is a direct sum of two of its subspaces W_1 and W_2 if and only if $V = W_1 + W_2$ and $W_1 \cap W_2 = \{0\}$. **(10)**

20. State and prove fundamental homomorphism theorem for vector spaces.

21. Prove that every finite dimensional inner product space V has an orthonormal set as a basis.

22. a) If T ∈ A(V) is Hermitian, then prove that all its eigen values are real.

b) Prove that T ∈A(V) is unitary if and only if takes an orthonormal basis of V on to an orthonormal basis of V. (6 +14)

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