LOYOLA COLLEGE (AUTONOMOUS), CHENNAI – 600 034

B.Sc. DEGREE EXAMINATION – **MATHEMATICS**

SIXTH SEMESTER – **APRIL 2018**

MT 6600 / MT 6603 – COMPLEX ANALYSIS

Date: 19-04-2018 Dept. No. Max. : 100 Marks

Time: 01:00-04:00

PART A

Answer all questions: **($10\times 2 = 20$)**

1. Prove that for any two complex numbers z_1 and z_2 $\left||z_1| - |z_2|\right| \leq |z_1 - z_2|$.
2. Show that $3x^2y + 2x^2 - y^3 - 2y^2$ is harmonic .
3. State Morera's theorem.
4. Evaluate $\int_C \frac{e^z}{z}$ where C is the unit circle $|z| = 1$.
5. Define zeros and poles of a function.
6. Find the zeros of $f(z) = \frac{z^2+1}{1-z^2}$.
7. Define residue of a function at a point.
8. State Argument theorem.
9. Define angle of rotation.
10. Define critical point.

PART B

Answer any five questions: **($5\times 8 = 40$)**

11. Let $f(z) = \begin{cases} \frac{xy}{x^2-y^2} & \text{if } z \neq 0 \\ 0 & \text{if } z = 0 \end{cases}$. Show that f(z) satisfies CR equations at zero but not differential at $z = 0$.
12. Prove that $u = 2x - x^3 + 3xy^2$ is harmonic and find its harmonic conjugate.
13. Find the radius of convergence of the power series (i) $\sum_{n=1}^{\infty} \frac{z^n}{n^2}$ (ii) $\sum_{n=1}^{\infty} \frac{z^n}{n}$.
14. State and prove Cauchy integral formula.
15. Find the Taylors series to represent $\frac{z-1}{z+1}$ in (i) z=0 (ii) z=1 .
16. State Maximum Modulus theorem.

17. Find the residue of the function $\frac{e^z}{z^2(z^2+9)}$ at its poles .

18. Define bilinear transformation and show that the transformation w = $\frac{5-4z}{4z-2}$ maps the unit circle $|z| = 1$ into a circle of radius unity and centre - $\frac{1}{2}$.

PART C

Answer any two questions: $(2\times 20 = 40)$

19 (a) Derive CR equations in polar coordinates . **(12+8)**

(b) Prove that functions f(z) and $\overline{f(z)}$ are simultaneously analytic.

20 (a) State and prove Cauchy's theorem and show that f'(z) = $\frac{r}{2}\left(\frac{\partial u}{\partial r} + i\frac{\partial v}{\partial r}\right)$.**(12+8)**

(b) State and prove Liouville's theorem and deduce fundamental theorem of algebra.

21 (a) Expand $f(z) = \frac{-1}{(z-1)(z-2)}$ in a Laurent's series in

(i) $1 < |z| < 2$, (ii) $|z| > 2$. **(12+8)** (b)Suppose

$f(z)$ is analytic in the region D and is not identically zero in D.Show that theset of all zeros of $f(z)$ is isolated .

22 (a) Using method of contour integration evaluate $\int_{-\infty}^{\infty} \frac{x^2}{(x^2+1)(x^2+4)}$dx .

(b) Find the bilinear transformation which maps the points -1 , 0 1 of z plane onto -1 , -i , 1 of the w plane. Show that under this transformation upper half of the z plane onto the interior of the unit circle $|w| = 1$.

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