LOYOLA COLLEGE (AUTONOMOUS), CHENNAI – 600 034

B.Sc. DEGREE EXAMINATION – **MATHEMATICS** [*******]

SIXTH SEMESTER – **APRIL 2018**

MT 6608– DISCRETE MATHEMATICS

Date: 21-04-2018 Dept. No. Max. : 100 Marks

Time: 09:00-12:00

PART-A

Answer all the questions **(10 x 2=20)**

1. Construct the truth table for $P \wedge (Q \wedge P)$.
2. What is the dual of $\neg(P \wedge Q) \vee \left(P \wedge \neg\left(Q \vee \neg S\right)\right)$.
3. Write down the min terms of P, Q and R.
4. Obtain the principle disjunctive normal form of $P \vee Q$
5. Define semigroup homomorphism.
6. Define submonoid.
7. Define lattice.
8. Define order isomorphic.
9. Define direct product of two Boolean algebras.
10. Define Boolean function.

PART-B

Answer any FIVE questions **(5 x 8=40)**

11. Construct the truth table for $\neg(P \wedge Q) \square (\neg P \vee \neg Q)$.
12. Show that $\left(\neg P \wedge \left(\neg Q \wedge R\right)\right) \vee (Q \wedge R) \vee (P \wedge R) \Leftrightarrow R$.
13. Show that $S \vee R$ is tautologically implied by $(P \vee Q) \wedge (P \rightarrow R) \wedge (Q \rightarrow S)$.
14. Show that $\neg\neg(P \wedge Q)$ follows from $\neg P \wedge \neg Q$.
15. Prove that for any commutative monoid $(M, *)$, the set of all idempotent elements of M forms a submonoid.
16. Let $(L, \leq)$ be a lattice in which $*$ and $\oplus$ denote the operations of meet and join respectively. Then prove that for any $a, b \in L$, $a \leq b \Leftrightarrow a * b = a \Leftrightarrow a \oplus b = b$.
17. Prove that the distributive inequalities hold in a lattice.
18. Obtain the values of the Boolean forms (a) $x_1 * x_2$ (b) $x_1 * (x_1' \oplus x_2)$ (c) $x_1 \oplus (x_1 * x_2)$

PART-C

Answer any TWO questions **(2 x 20=40)**

19. (a) Show that $\left((P \vee Q) \wedge \neg\left(\neg P \wedge (\neg Q \wedge \neg R)\right)\right) \vee (\neg P \wedge \neg Q) \vee (\neg P \wedge \neg R)$ is a tautology.

(b) Prove that if $H_1, H_2, H_3, ..., H_m$ and P imply Q, then $H_1, H_2, H_3, ..., H_m$ imply $P \to Q$.

(10+10)

20. (a) Show that the following premises are inconsistent.

I If Jack misses many classes through illness, then he fails in high school.

II If Jack fails high school, then he is uneducated.

III If Jack reads a lot of books, then he is not uneducated.

IV Jack misses many classes through illness and reads a lot of books.

(b) Prove that the composition of semigroup homomorphism is also a semigroup homomorphism. **(10+10)**

21. (a) State and prove any four properties of lattice.

(b) Define Boolean algebra and give an example. **(16+4)**

22. (a) Write down the following Boolean expressions in an equivalent sum of the product of canonical forms in three variables x_1, x_2 and x_3

(i) $x_1 * x_2$ (ii) $x_1 \oplus x_2$ (iii) $(x_1 \oplus x_2)' * x_3$.

(b) Define the following (i) complete lattice (ii) complemented lattice (iii) distributive lattice (iv) bounded lattice.

(12+8)

****