LOYOLA COLLEGE (AUTONOMOUS), CHENNAI – 600 034

B.Sc.DEGREE EXAMINATION – **MATHEMATICS**

FOURTH SEMESTER – **APRIL 2019**

16/17UMT4MC01– ABSTRACT ALGEBRA

Date: 03-04-2019 Dept. No. Max. : 100 Marks

Time: 09:00-12:00

Part A

Answer All Questions: **(10 x 2 = 20)**

1. For all $a, b \in G$, show that $(a \cdot b)^{-1} = b^{-1} \cdot a^{-1}$.

2. If G is a group with $o(G) = 12$, find all the possible orders of the elements of G.

3. Show that every subgroup of an abelian group is normal.

4. Define quotient group.

5. If G is a group of real numbers under addition and $\bar{G}$ is a group of real numbers under multiplication and ¤ $: G \to \bar{G}$ is defined as ¤$(x) = 2^x, x \in G,$ check whether ¤ is a homomorphism or not.

6. Write the cycles of

$$\begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 \\ 2 & 3 & 4 & 5 & 1 & 6 & 7 & 9 & 8 \end{pmatrix}$$

7. What is a division ring?

8. When is an integral domain said to be of characteristic zero?

9. If U is an ideal of R and if $1 \in U$, show that $R = U$.

10. Define Euclidean ring.

Part B

Answer Any Five Questions: **(5x8 = 40)**

11.State and prove the necessary and sufficient condition for a nonempty subset of a group to be a subgroup of the group.

12. If G is a group, show that for all $a \in G$, $H_a = \{x \in G / a \equiv x \bmod H\}$.

13. If H and K are subgroups of a group G, prove that HK is a subgroup of G if and only if $HK = KH$.

14. Show that the subgroup N of a group G is a normal subgroup of G if and only if every left coset of N in G is a right coset of N in G.

15. If ¤ is a homomorphism of a group G into another group $\bar{G}$ with kernel K, prove that$K$ is a normal subgroup of G.

16. Prove that the homomorphism φ of a ring R into a ring R′ is a isomorphism if and only if I(φ) = {0}.

17. If R is a commutative ring with unit element whose only ideals are (0) and Ritself, prove that R is a field.

18. Let A be an ideal of a Euclidean ring R. Prove that there exists an element $a_0 \in$A such that A consists exactly of all $a_0 x$ as x ranges over R.

Part C

Answer any Two Questions: **(2x20 = 40)**

19. State and prove Lagrange's Theorem with necessary lemmas. **(20)**

20. (a) If ¤ is a homomorphism of a group G onto a group $\overline{G}$ with kernel K, then prove that G/K$\cong \overline{G}$.

(b) State and prove Cayley's theorem. **(10+10)**

21. (a)Prove that the set of integers mod 7 under addition and multiplication mod 7 is a ring.

(b)Prove that a finite integral domain is a field. **(12+8)**

22. (a) If U is an ideal of a ring R, prove that R/U is a ring and is a homomorphic image of$R$.

(b) State and prove unique factorization theorem. **(10+10)**

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