LOYOLA COLLEGE (AUTONOMOUS), CHENNAI – 600 034

B.Sc. DEGREE EXAMINATION – **MATHEMATICS**

SIXTH SEMESTER – **APRIL 2019**

16UMT6MC03– DISCRETE MATHEMATICS

Date: 10-04-2019 Dept. No. Max. : 100 Marks

Time: 09:00-12:00

SECTION – A

ANSWER ALL QUESTIONS **(10 X 2 = 20 Marks)**

1. Define conjunction with truth table.
2. Construct truth table for the statement formula P $\vee$ ⌉Q.
3. Write down the Rules of Inference.
4. Define Principal Disjunctive Normal Form of a statement.
5. Define monoid with an example.
6. Define sub semigroup.
7. Define partially ordered set.
8. State isotonic property in lattice.
9. Define Boolean algebra.
10. Define Boolean homomorphism.

SECTION – B

ANSWER ANY FIVE QUESTIONS **(5 x 8 = 40 Marks)**

11. Show that (⌉P $\wedge$ (⌉Q$\wedge$R)) $\vee$ (Q$\wedge$R) $\vee$ (P$\wedge$R) $\Leftrightarrow$ R.
12. (a) Prove that $P \rightarrow Q \Leftrightarrow$ ⌉Q $\rightarrow$ ⌉P

    (b) Show that (*P* e *Q*) ė ķ(*P* Ę *Q*) is a contradiction
13. Obtain DNF for the statement ķ(*P*Ę *Q*)⇄(*P* ė *Q*) .
14. Let $(S,*)$ be a semigroup and R be a congruence relation on $(S,*)$. Then prove that the quotient group S/R is a semigroup $(S/R,\oplus)$ where the operation $\oplus$ corresponds to the operation $*$on S. Also prove that there exists a homomorphism from $(S,*)$ onto $(S/R,\oplus)$ called the natural homomorphism.
15. Prove that for any commutative monoid$(M,*)$, the set of idempotent elements of M forms a sub monoid.
16. State and prove the distributive inequalities in lattice.
17. Obtain sum-of-product canonical form for $(x \oplus y) * (x' \oplus z) * (y \oplus z)$
18. Prove that in a complemented distributive lattice, complement is unique.

SECTION -C

ANSWER ANY TWO QUESTIONS (2 X 20 = 40 Marks)

19. (a) Show that $((P \vee Q) \wedge \neg(\neg P \wedge (\neg Q \vee \neg R))) \vee (\neg P \wedge \neg Q) \vee (\neg P \wedge \neg R)$ is a tautology.

    (b) Construct truth table for $[(P \vee Q) \wedge (P \rightarrow R) \wedge (Q \rightarrow R)] \rightarrow R$ **(10 + 10)**

20. (a) Obtain the principal conjunctive normal form of $(\neg P \rightarrow R) \wedge (Q \rightleftarrows P)$.

    (b) Show that $S \vee R$ is tautologically implied by $(P \vee Q) \wedge (P \rightarrow R) \wedge (Q \rightarrow S)$.

    (10 + 10)

21. (a) Explain the following terms:

    (i) Direct product of two Boolean algebras (ii) Lattice Homomorphism

    (b) If $\langle L, \leq \rangle$ is a lattice, then for any a, b, c $\in$ L, prove that

    $a \leq b \Leftrightarrow a \wedge b' = 0 \Leftrightarrow a' \vee b = 1 \Leftrightarrow b' \leq a'$.

    (8+12)

22. (a) State and prove De Morgan's laws in lattices.

    (b) Prove that the composition of two semigroup homomorphism is also a semigroup homomorphism.

    (10 + 10)

★★★★★★