



Date: 11-04-2019

Dept. No.

Max. : 100 Marks

Time: 09:00-12:00

Part A

Answer any FOUR questions:

(4 x 10 = 40)

1. Show that the equations $x + 2y - z = 3$, $3x - y + 2z = 1$, $2x - 2y + 3z = 2$, $x - y + z = -1$. are consistent and solve them.
2. Prove that $\frac{\cos 7\theta}{\cos \theta} = 64 \cos^6 \theta - 112 \cos^4 \theta + 56 \cos^2 \theta - 7$.
3. Find the condition that the roots of the equation $ax^3 + 3bx^2 + 3cx + d = 0$ may be in geometric progression.
4. Verify Euler's theorem for the function $u = \sin^{-1} \left(\frac{x^2 + y^2}{x + y} \right)$.
5. Prove that $\int_0^{\frac{\pi}{2}} \frac{(\sin x)^{3/2}}{(\sin x)^{3/2} + (\cos x)^{3/2}} dx = \frac{\pi}{4}$
6. Solve the equation $(D^2 + 4D + 4)y = e^{-2x}$.
7. Solve $xp + yq = x$.
8. Determine the root of $xe^x - 3 = 0$ correct to three decimals using RegulaFalsi method.

Part B

Answer any THREE questions:

(3 x 20 = 60)

9. a) Prove that $\frac{\sin 7\theta}{\sin \theta} = 7 - 56 \sin^2 \theta + 112 \sin^4 \theta - 64 \sin^6 \theta$.
- b) Find the Eigen values and Eigen vectors of the matrix $\begin{pmatrix} 4 & -20 & -10 \\ -2 & 10 & 4 \\ 6 & -30 & -13 \end{pmatrix}$.
10. a) Prove that $\cos^5 \theta \sin^4 \theta = \frac{1}{2^8} [\cos 9\theta + \cos 7\theta - 4 \cos 5\theta - 4 \cos 3\theta + 6 \cos \theta]$.
- b) Solve the reciprocal equation $x^5 + 4x^4 + x^3 + x^2 + 4x + 1 = 0$.
11. a) Find the radius of curvature of the curve $xy^2 = a^3 - x^3$ at the points $(a, 0)$.
- b) Prove that $\int_0^{\frac{\pi}{4}} \log(1 + \tan \theta) d\theta = \frac{\pi}{8} \log 2$.
12. a) Solve the equation $(D^2 + 5D + 4)y = e^x$.
- b) Find the real root of $x^3 - 2x - 5 = 0$ by false position method correct to 3 decimal places.

13. a) The velocity of a particle at distance S from a point on it's path is given by the following table

S(ft)	0	10	20	30	40	50	60
V(ft/s)	47	58	64	65	61	52	38

Estimate the time taken to travel 60 ft using Trapezoidal rule.

b) Find all the characteristic roots and the associated characteristic vectors of the matrix

$$A = \begin{bmatrix} 8 & -6 & 2 \\ -6 & 7 & -4 \\ 2 & -4 & 3 \end{bmatrix}$$

14. a) Solve the reciprocal equation $6x^5 - x^4 - 43x^3 + 43x^2 + x - 6 = 0$.

b) If $u = \tan^{-1} \left(\frac{x^3 + y^3}{x + y} \right)$, prove that $x \frac{\partial u}{\partial x} + y \frac{\partial u}{\partial y} = \sin 2u$.

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