

LOYOLA COLLEGE (AUTONOMOUS), CHENNAI – 600 034



B.Sc. DEGREE EXAMINATION – MATHEMATICS

SIXTH SEMESTER – APRIL 2022

16/17UMT6MC01 – COMPLEX ANALYSIS

Date: 15-06-2022

Dept. No.

Max. : 100 Marks

Time: 01:00 PM - 04:00 PM

PART – A

Answer ALL Questions

(10 x 2 = 20)

1. Define harmonic function.
2. Verify Cauchy-Riemann equations for $f(z) = e^z$.
3. Evaluate $\int_C \frac{dz}{z-4}$ where C is the circle $|z| = 3$.
4. Define simply connected region.
5. Write the Maclaurin series for $f(z) = e^{-z}$.
6. State Laurent's theorem.
7. Classify the singularity of $f(z) = \frac{1}{1-z}$.
8. Find the residue of $f(z) = \frac{z+1}{z+9}$ at its pole.
9. Find the fixed point of $w = \frac{z-1}{z+1}$.
10. Define conformal mapping.

PART – B

Answer any FIVE Questions

(5 x 8 = 40)

11. Prove that an analytic function with constant modulus is constant.
12. State and prove the Cauchy integral formula.
13. State and prove fundamental theorem of algebra.
14. State and prove Cauchy residue theorem.
15. Find the linear fractional transformation that maps the points $z_1 = -1$, $z_2 = 0$ and $z_3 = 1$ onto $w_1 = -i$, $w_2 = 1$ and $w_3 = i$.
16. State and prove the argument principle.
17. Evaluate $\int_0^{2\pi} \frac{d\theta}{5+4 \sin \theta}$ using residues.
18. Determine the number of zeros counting multiplicities of the polynomial $z^6 - 5z^4 + z^3 - 2z$ inside the circle $|z| = 1$.

PART – C

Answer any TWO Questions

(2 x 20 = 40)

19. State and prove the necessary and sufficient conditions for differentiability of a function.

20. State and prove Cauchy-Goursat theorem.

21. a). State and prove Taylor's theorem.

b). State and prove Liouville's theorem.

(14+6)

22. Using residues, evaluate $\int_0^{\infty} \frac{x^2}{x^6+1} dx$.

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