LOYOLA COLLEGE (AUTONOMOUS), CHENNAI – 600 034

B.Sc. DEGREE EXAMINATION – **PHYSICS**

SECOND SEMESTER – **APRIL 2022**

16/17/17UMT2AL01 – MATHEMATICS FOR PHYSICS - II

Date: 27-06-2022 Dept. No. Max. : 100 Marks

Time: 01:00 PM - 04:00 PM

Part A

Answer ALL the questions **(10 × 2 = 20)**

1. Evaluate $\int(ax^2+bx+c)dx$.
2. Find the value of $\int_0^{\frac{\pi}{2}} cos^6 x dx$.
3. Prove that $\int_0^a f(x)dx = \int_0^a f(a-x)dx$.
4. Prove that $\beta(m,n) = \beta(n,m)$.
5. Solve $\frac{dy}{dx} + ycotx = cosex$.
6. If the roots are real and distinct that is α and β, then what is complementary function?
7. Evaluate $\int_0^a \int_0^b xy(x-y)dydx$.
8. Find $\frac{\partial(x,y)}{\partial(r,\theta)}$ when $x = rcos\theta$ and $y = rsin\theta$.
9. Find the directional derivative of $\varphi = x^2yz + 4xz^2$ at (1,-2,-1) in the direction of $2\vec{i} - \vec{j} - 2\vec{k}$.
10. Find 'a' such that $(3x-2y+z)\vec{i} + (4x+ay-z)\vec{j} + (x-y+2z)\vec{k}$ is solenoidal.

Part B

Answer any FIVE questions **(5 × 8 = 40)**

11. Evaluate $\int \frac{(3x+1)}{(x-1)^3(x+3)} dx$.
12. Establish the reduction formula for $I_n = \int tan^n x\, dx$ (n being a positive integer) and hence find the value of $\int_0^{\frac{\pi}{4}} tan^3 x\, dx$.
13. Solve $I = \int_0^{\frac{\pi}{2}} \log sinx\, dx$.
14. Show that $\frac{2^n \Gamma(n+\frac{1}{2})}{\sqrt{\pi}} = 1.3.5 \ldots\ldots (2n-1)$.
15. Solve $\frac{d^2y}{dx^2} + 4\frac{dy}{dx} + 4y = e^{-x} sin2x$.
16. Find $x\frac{dy}{dx} + ylogx = e^x x^{1-\frac{1}{2}logx}$.
17. Evaluate $\int_0^{\frac{\pi}{2}} \int_{a(1-cos\theta)}^{a} r^2 dr d\theta$.
18. Using Green's theorem, evaluate $\int_C \{(3x-8y^2)dx + (4y-6xy)dy\}$ where C is the boundary of the region given by $x=0, y=0, x+y=1$.

Part C

Answer any TWO questions **(2 × 20 = 20)**

19. (a) Derive the reduction formula for $I_n = \int sin^n x\, dx$.
    (b) Solve $(D^2 - 6D + 25)y = e^{2x} + \sin x + x$. **(10+10)**

20. (a) Prove that $\beta(m,n) = \frac{\Gamma(m)\Gamma(n)}{\Gamma(m+n)}$.
    (b) Show that $\Gamma\left(\frac{1}{2}\right) = \sqrt{\pi}$. **(15+5)**

21. (a) Change the order of integration and hence evaluate $\int_0^b \int_0^{\frac{a}{b}\sqrt{b^2-y^2}} xydxdy$.

(b) Evaluate $\int_0^\infty \int_0^\infty e^{-(x^2+y^2)}dxdy$. **(15+5)**

22. (a) Verify Gauss Divergence theorem for $\vec{F} = 4xz\vec{\imath} - y^2\vec{\jmath} + yz\vec{k}$ over the cube bounded by $x = 0, x = 1, y = 0, y = 1, z = 0, z = 1$.

(b) If $\vec{F} = x^2y\vec{\imath} + y^2z\vec{\jmath} + z^2x\vec{k}$, then find $curl\ (curl\vec{F})$. **(15+5)**

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