LOYOLA COLLEGE (AUTONOMOUS), CHENNAI – 600 034

B.Sc. DEGREE EXAMINATION – **STATISTICS**

SECOND SEMESTER – **APRIL 2022**

16/17/18/UMT2AL02 – MATHEMATICS FOR STATISTICS- II

Date: 27-06-2022 Dept. No. Max. : 100 Marks

Time: 01:00 PM - 04:00 PM

SECTION A

ANSWER ALL QUESTIONS. **(10 × 2 = 20)**

1. Define least upper bound.
2. Define limit of a sequence.
3. Prove that if $a_1 + a_2 + \cdots$ converges to s, then $a_2 + a_3 + \cdots$ converges to $s - a_1$.
4. Give an example of divergent series.
5. Define the right hand limit of *f* at *a*.
6. When you say the subset *D* of *R* is of first category?
7. Find a suitable point c of Rolle's theorem for $f(x) = (x-a)(b-x), (a \le x \le b)$.
8. Prove that the derivative of a constant function on $[a, b]$ is the identically zero function on $[a, b]$.
9. Define upper sum $U[f: \sigma]$ of *f* corresponding to partition σ.
10. What is meant by a subdivision of the closed bounded interval $[a, b]$?

SECTION B

ANSWER ANY FIVE QUESTIONS. **(5 × 8 = 40)**

11. If *A, B* are subsets of *S*, then prove that

    (i) $(A \cup B)' = A' \cap B'$ and (ii) $(A \cap B)' = A' \cup B'$.
12. Evaluate the following: (i) $\lim_{n\to\infty} \frac{2n^3+5n}{4n^3+n^2}$ and (ii) $\lim_{n\to\infty} \frac{n^2}{(n-7)^2-6}$. **(4+4)**
13. If $\sum_{n=1}^{\infty} a_n$ is a convergent series, then prove that $\lim_{n\to\infty} a_n = 0$.
14. Prove that the product of two continuous function is continuous.
15. If f and g are continuous real valued functions at a and $f(a)$ respectively, then prove that $g \circ f$ is continuous at a.
16. Let $f: R^1 \to R^1$. For any $r > 0$ let E_r be the set of all $a \in R^1$ such that $\omega[f; a] \ge \frac{1}{r}$ then prove that E_r is closed.
17. State and prove the Law of mean.
18. Let *f* be a bounded function on the closed bounded interval $[a, b]$, then prove that $f \in \mathfrak{R}[a, b]$ if and only if for each $\varepsilon > 0$, there exists a subdivision σ of $[a, b]$ such that $U[f: \sigma] < L[f: \sigma] + \varepsilon$.

SECTION C

ANSWER ANY TWO QUESTIONS. **(2 x 20 = 40)**

19. (a) Define bounded sequence and hence prove that if the sequence of real numbers $\{s_n\}_{n=1}^{\infty}$ is convergent then $\{s_n\}_{n=1}^{\infty}$ is bounded.

    (b) Prove that $\lim_{n\to\infty} \frac{3n^2-6n}{5n^2+4} = \frac{3}{5}$. **(12+8)**

20. (a) State and prove Comparison test for series.

    (b) Prove that the series $\sum_{n=1}^{\infty} \frac{1}{n}$ is divergent. **(10+10)**

21. State and prove Taylor's theorem. **(20)**

22. (a) If $f, g \in \Re[a, b]$, then prove that $f + g \in \Re[a, b]$ and $\int_a^b (f + g) = \int_a^b f + \int_a^b g$.

    (b) Let f be a bounded function on $[a, b]$. If σ and τ are any two subdivisions of $[a, b]$, then prove that $U[f:\sigma] \geq L[f:\tau]$. **(12+8)**

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