LOYOLA COLLEGE (AUTONOMOUS), CHENNAI – 600 034

B.Sc. DEGREE EXAMINATION – **MATHEMATICS**

SECOND SEMESTER – **APRIL 2022**

16/17/18UMT2MC01 – ALGEBRA AND CALCULUS - II

Date: 16-06-2022 Dept. No. Max. 100 Marks

Time: 01:00-04:00

PART – A

Answer ALL questions **(10 x 2 = 20)**

1. Evaluate $\int sin^2 x \ \ dx$.
2. If $f(x)$ is an odd function, show that $\int_{-a}^{a} f(x)dx = 0$.
3. Evaluate $\int_0^1 \int_0^1 xy\ dxdy$.
4. Find the Jacobian $J\left(\frac{x,y}{r,\theta}\right)$, if $x = r\cos\theta$ and $y = r\sin\theta$.
5. Define Gamma function.
6. Prove that $\beta(m,n) = \beta(n,m)$, for $m,n \in \mathbb{N}$.
7. Show that the sequence $\left\{\frac{n}{n+1}\right\}$ is monotonic
8. State Raabe's test.
9. Expand $(1-x)^{-1}$ for $|x| < 1$.
10. Expand $\frac{e^x+e^{-x}}{2}$.

PART – B

Answer any FIVE questions **(5 x 8 = 40)**

11. Show that $\int_0^{\pi/2} \frac{\sqrt{\sin x}}{\sqrt{\sin x} + \sqrt{\cos x}} dx = \frac{\pi}{4}$.
12. Find the area of the cardioid $r = a(1+\cos\theta)$.
13. Evaluate $\iint xy\ dxdy$ taken over the positive quadrant of the circle $x^2 + y^2 = a^2$.
14. Evaluate $\int_0^{\infty} e^{-x^2}\ dx$ using Gamma integrals.
15. Sum the series $\sum_{n=0}^{\infty} \frac{5n+1}{(2n+1)!}$.
16. Prove that $\log\sqrt{12} = \left(\frac{1}{2}+\frac{1}{3}\right)\frac{1}{4} + \left(\frac{1}{4}+\frac{1}{5}\right)\frac{1}{4^2} + \cdots$
17. Test the convergence of the series $\sum_{n=1}^{\infty} \left(\frac{n}{n+1}\right)^{1/2} x^n$.
18. Show that $\frac{\frac{1}{2!}+\frac{1}{4!}+\frac{1}{6!}+\cdots}{\frac{1}{1!}+\frac{1}{3!}+\frac{1}{5!}+\cdots} = \frac{e-1}{e+1}$.

PART – C

Answer any TWO questions **(2 x 20 = 40)**

19. a). Evaluate $\int_0^{\pi/2} \log \sin x \, dx$.

    b). Find the reduction formula for $I_n = \int (\sin x)^n \, dx$. **(10+10)**

20. Change the order of integration and evaluate $\int_0^a \int_{x^2/a}^{2a-x} xy \, dydx$.

21. Prove that $\beta(m,n) = \frac{\Gamma(m)\Gamma(n)}{\Gamma(m+n)}$.

22. a). Find the sum of the series $1 + \frac{1+3}{2!} + \frac{1+3+3^2}{3!} + \frac{1+3+3^2+3^3}{4!} + \cdots$

    b). Show that $\frac{5}{1\cdot2\cdot3} + \frac{7}{3\cdot4\cdot5} + \frac{9}{5\cdot6\cdot7} + \cdots = 3\log 2 - 1$. **(10+10)**

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