

LOYOLA COLLEGE (AUTONOMOUS), CHENNAI – 600 034



B.Sc. DEGREE EXAMINATION – MATHEMATICS

SECOND SEMESTER – APRIL 2022

16/17/18UMT2MC02 – ANA. GEO. OF 3D, FOURIER SERIES AND NUMBER THEORY

Date: 18-06-2022

Dept. No.

Max. : 100 Marks

Time: 01:00-04:00

PART – A

Answer ALL the questions:

(10 × 2 = 20)

1. Write the equation of the symmetric form of the straight line.
2. Give the general equation of a straight line.
3. Find the equation of the sphere whose centre is (1,2,3) and radius is 4.
4. Write the equation of the tangent plane to a sphere.
5. Give the expression for a_0 in Fourier series.
6. List out Dirichlet's criterion.
7. Determine the number of divisors of 360.
8. Find the number of integers less than 729 and prime to it.
9. Show that $n^n > 1.3.5 \dots (2n - 1)$.
10. State Cauchy's inequality.

PART – B

Answer any FIVE of the following:

(5 × 8 = 40)

11. Find the equation of the sphere through the points (0, -2,3), (1, 5, -1), (2, 0, 1) and (4, -1, 2).
12. Determine the shortest distance between the lines $\frac{x-3}{-1} = \frac{y-4}{2} = \frac{z+2}{1}$ and $\frac{x-1}{1} = \frac{y+7}{3} = \frac{z+2}{2}$.
13. Find a sine series for $f(x) = c$ in the range $(0, \pi)$.
14. Express $f(x) = \frac{1}{2}(\pi - x)$ as a Fourier series with period 2π , to be valid in the interval $(0, 2\pi)$.
15. Find the highest power of 3 dividing 1000!.
16. Find the remainder obtained in dividing 2^{46} by 47.
17. Show that $(x^m + y^m)^n < (x^n + y^n)^m$ if $m > n$.
18. State and prove Weierstrass inequality.

PART – C

Answer any TWO of the following:

(2 × 20 = 40)

19. a) Find the equation of the plane through (2, -1, 1) and perpendicular to the line joining the points (3,4, -1) and (2, -1,5).
b) The plane $\frac{x}{a} + \frac{y}{b} + \frac{z}{c} = 1$ meets the axes in A, B, C. Find the equation of the circumcircle of the triangle ABC and determine also the coordinates of the centre and radius.
20. Develop a Fourier series expansion for $f(x) = \frac{x^2}{4}$ in the interval $[-\pi, \pi]$. Deduce the following. (a) $\frac{1}{1^2} + \frac{1}{2^2} + \frac{1}{3^2} + \dots = \frac{\pi^2}{12}$ (b) $\frac{1}{1^2} + \frac{1}{2^2} + \frac{1}{3^2} + \dots = \frac{\pi^2}{6}$.
21. a) Show that $13^{2n+1} + 9^{2n+1}$ is divisible by 22.
b) Show that 8th power of any number is of the form $17m$ or $17m \pm 1$.
22. a) State and prove Wilson's theorem.
b) State and prove Fermat's theorem.

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