LOYOLA COLLEGE (AUTONOMOUS), CHENNAI – 600 034

B.Sc. DEGREE EXAMINATION – **MATHEMATICS**

FOURTH SEMESTER – **APRIL 2022**

16/17/18UMT4ES01 – COMBINATORICS

Date: 23-06-2022   Dept. No. [ ]   Max. : 100 Marks

Time: 09:00 AM - 12:00 NOON

PART – A

Answer ALL the Questions. **(10 x 2 = 20)**

1. If $a_n = na_{n-1}$ and $a_0 = 1$. Find a_3
2. Expand $(1 + x)^6$.
3. Construct 2 different 5 x 5 Latin squares, which have the same first rows and other rows are different.
4. Write down all the possible derangements of 1234.
5. In how many ways seven men are to be seated around a circular table?
6. Define a generating function.
7. How many possibilities are there for 10 people to be seated in 4 sears?
8. Define a tree.
9. Define the inclusion and exclusion principle.
10. If $A_1 = \{1,2\}$, $A_2 = \{4\}$, $A_3 = \{1,3\}$ and $A_4 = \{2,3,4\}$. Find the distinct representatives for the sets A_i.

PART –B

Answer any FIVE of the following Questions. **(5 x 8 = 40)**

11. Suppose that $f(n, n-1) = 1$ and $(n-k-1)f(n,k) = k(n-1)f(n,k+1)$ for each $k < n-1$. Prove that $f(n,k) = \frac{(n-1)^{n-k-1}(n-2)!}{(k-1)!(n-k-1)!}$.
12. Prove that if a graph has $2n$ vertices, each of degree $\geq n$, then the graph has a perfect matching.
13. State and prove the Landau's theorem.
14. Find a_n if $a_n = 4\,a_{n-1} + 4\,a_{n-2} - 16\,a_{n-3}$, $a_1 = 8$, $a_2 = 4$, $a_3 = 24$.
15. Let S be a set of mn objects. Then show that S can be partitioned into n sets of m elements in $\frac{(mn)!}{(m)!^n n!}$.
16. Explain Ordered Selection and evaluate the following: a) $p(7,4)$, b) $p(9,5)$
17. Explain the Inclusion and Exclusion Principle with an example.
18. a) How many permutations are there of the 26 letters of the alphabet in which the 5 vowels are in consecutive places. **(4+4)**

    b) How many different necklaces can be designed from n colors, using one bead of each color?

PART – C

Answer any <u>TWO</u> of the following Questions. (2 x 20 = 40)

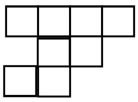
19. a) State and prove marriage theorem.

b) Solve the Fibonacci – type relations. **(10+10)**

20. a) Find the optimal assignment for the following problem.

	MAN				
		A	B	C	D
JOB	a	6	8	2	7
	b	5	8	13	9
	c	2	7	8	9
	d	4	11	7	10

b) Prove that if a graph has $2n$ vertices, each of degree $\geq n$, then the graph has a perfect matching. **(10+10)**

21.Find the rook polynomial of the board 

22. a) State and prove the exchange property.

b) Let n be a positive integer, show that if $(1+x)^n$ is expanded as a sum of powers of n, the coefficient of x^r is $\binom{n}{r}$. **(10+10)**

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