LOYOLA COLLEGE (AUTONOMOUS), CHENNAI – 600 034

M.Sc. DEGREE EXAMINATION – **MATHEMATICS**

SECOND SEMESTER – **APRIL 2022**

PMT 2503 – PARTIAL DIFFERENTIAL EQUATIONS

Date: 17-06-2022 Dept. No. Max. : 100 Marks

Time: 09:00 AM - 12:00 NOON

ANSWER ALL QUESTIONS

1. (a) Find the partial differential equation of the family of planes whose sum of x, y, z intercepts is equal to unity. **(5)**

(OR)

(b) Solve: $\begin{vmatrix} x & y & z \\ \alpha & \beta & \gamma \\ \frac{\partial z}{\partial x} & \frac{\partial z}{\partial y} & -1 \end{vmatrix} = 0$ **(5)**

(c) When do you say that two first order partial differential equations are compatible? Check whether the following partial differential equations $xp - yq = x$ and $x^2p + q = xz$ are compatible and if so, find their solution. **(15)**

(OR)

(d) (i) Eliminate the arbitrary function from $F(x + y + z, x^2 + y^2 + z^2) = 0$. **(5)**

(ii) Find the characteristics of the equation $pq = z$ and hence determine the integral surface which passes through the parabola $x = 0,\ y^2 = z$. **(10)**

2. (a) Find the adjoint operators for $L(u) = u_{xx} + u_{yy}$ and $L(u) = u_{xx} - u_t$. **(5)**

(OR)

(b) Determine the suitable characteristics for $y^2u_{xx} - 2xyu_{xy} + x^2u_{yy} = \frac{y^2}{x}u_x + \frac{x^2}{y}u_y$. **(5)**

(c) Reduce $u_{xx} - 2sinx u_{xy} - cos^2x\, u_{yy} - cosx\, u_y = 0$ to a canonical form and hence solve it. **(15)**

(OR)

(d) Explain the Riemann's method for solving the hyperbolic partial differential equation. **(15)**

3. (a) Find the solution of Laplace's equation in cylindrical coordinates. **(5)**

(OR)

(b) Examine whether the following provides a non-trivial solution to $\nabla^2 u = 0$ under the boundary conditions $u(x, b) = u(a, y) = u(0, y) = 0$.

(1) $u(x, y) = (c_1e^{px} + c_2e^{-px})(c_3cospy + c_4sinpy)$

(2) $u(x,y) = (c_1 x + c_2)(c_3 y + c_4)$ **(5)**

(c) Solve $\nabla^2 u = 0$, $0 \le r \le a$, $0 \le \theta \le 2\pi$, under the boundary conditions $u(a,\theta) = f(\theta)$. **(15)**

(OR)

(d) Solve the Neumann's problem for a rectangle. **(15)**

4. (a) Find the periodic solution of one-dimensional wave equation in cylindrical coordinates. **(5)**

(OR)

(b) Prove that $u(x,t) = \frac{1}{2}[\eta(x+ct) + \eta(x-ct)] + \frac{1}{2c}\int_{x-ct}^{x+ct} v(\xi)d\xi$ is the solution of the equation $u_{tt} = c^2 u_{xx}$, $-\infty < x < \infty$, $t \ge 0$, with initial conditions $u(x,0) = \eta(x)$, $u_t(x,0) = v(x)$. **(5)**

(c) Solve the following wave equation: $u_{tt} = c^2 u_{xx}$, $0 \le x \le L$, $t > 0$,

$u(0,t) = u(L,t) = 0$, $u(x,0) = f(x)$, $u_t(x,0) = g(x)$. **(15)**

(OR)

(d) Solve the one-dimensional diffusion equation in the region $0 \le x \le \pi$, $t \ge 0$, subject to the conditions (i) T remains finite as $t \to \infty$, (ii) $T = 0$, if $x = 0$ and π for all t, (iii) At $t = 0$,

$$T = \begin{cases} x & : 0 \le x \le \pi/2 \\ \pi - x & : \pi/2 \le x \le \pi \end{cases}$$ **(15)**

5. (a) Prove that the Green's function has the symmetric property. **(5)**

(OR)

(b) Use Green's function technique to solve the Dirichlet's problem for a semi-infinite space. **(5)**

(c) Using the Laplace transform method, solve $u_{xx} = \frac{1}{c^2} u_{tt} - cos\omega t$, $0 \le x < \infty$, $0 \le t < \infty$, subject to $u(0,t) = 0$, u is bounded as $x \to \infty$, $u_t(x,0) = u(x,0) = 0$. **(15)**

(OR)

(d) State and prove Helmholtz theorem. **(15)**

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