LOYOLA COLLEGE (AUTONOMOUS), CHENNAI – 600 034

M.Sc. DEGREE EXAMINATION – **MATHEMATICS**

SECOND SEMESTER – **APRIL 2022**

PMT 2504 – COMPLEX ANALYSIS

Date: 22-06-2022 Dept. No. Max. : 100 Marks

Time: 09:00-12:00

Answer all Questions. All questions carry equal marks.

1. a. State and prove Cauchy's integral formula and hence evaluate

$\int_{\gamma} \frac{sinz}{z} dz$ where $\gamma(t) = e^{it}, 0 \leq t \leq 2\pi$.

OR

b. State and prove the Fundamental theorem of Algebra. (5)

c. State and prove the homotopic version of Cauchy's theorem.

OR

d. Prove that any differentiable complex valued function defined on an open set is analytic.

(15)

2. a. If $\gamma: [0,1] \to \mathfrak{C}$ is a closed rectifiable curve and $a \notin \{\gamma\}$, then prove that the winding number of γ around a is an integer.

OR

b. State and prove Morera's theorem. (5)

c. State and prove Hadamard's three circles theorem.

OR

d. Prove that all proper simply connected regions are conformally equivalent to the open unit disk. (15)

3. a. Prove that $sin\pi z = \pi z \prod_{n=1}^{\infty} \left(1 - \frac{z^2}{n^2}\right)$.

OR

b. Prove that $\gamma(z + 1) = \gamma(z)$ for $z \neq 0, -1, ...$ (5)

c. State and prove the Weierstrass factorization theorem.

OR

d. State and prove Bohr-Mollerup theorem. (15)

4. a. State and prove Euler's theorem.

OR

b. State and prove Jensen's formula. (5)

c. State and prove Hadamard's factorization theorem.

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OR

d. If f is an entire function of finite genus μ, then prove that f is of finite order $\lambda \leq \mu + 1$.

(15)

5. a. Prove that the zeros $a_1, a_2 \dots a_n$ and poles $b_1, b_2 \dots b_n$ of an elliptic function $f(z)$ satisfy $a_1 + a_2 \dots + a_n \equiv b_1 + \ b_2 \dots + b_n (mod\ M)$, where M is the period module of $f(z)$.

OR

b. Prove that any two bases of the same module are connected by a unimodular transformation. (5)

c. i) Prove that $\wp(z) - \wp(u) = -\frac{\sigma(z-u)\sigma(z+u)}{\sigma(z)^2\sigma(u)^2}$ (7)

ii) Prove that $\xi(z+u) = \xi(z) + \xi(u) + \frac{1}{2}\frac{\wp'(z)-\wp'(u)}{\wp(z)-\wp(u)}$. (8)

OR

d. i) Show that any even elliptic function with periods w_1 and w_2 can be expressed in the form $C\prod_{k=1}^{n}\frac{\wp(z)-\wp(a_k)}{\wp(z)-\wp(b_k)}$ where C is constant. (7)

ii) Show that any elliptic function with periods w_1 and w_2 can be expressed in the form $C\prod_{k=1}^{n}\frac{\sigma(z-a_k)}{\sigma(z-b_k)}$ where C is constant. (8)

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