LOYOLA COLLEGE (AUTONOMOUS), CHENNAI – 600 034

B.Sc. DEGREE EXAMINATION – **MATHEMATICS**

FIRST SEMESTER – **APRIL 2022**

UMT 1502 – CALCULUS

(21 BATCH ONLY)

Date: 18-06-2022 Dept. No. Max. : 100 Marks

Time: 09:00 AM - 12:00 NOON

SECTION A

Answer ALL the Questions

1.	**Answer the following**		**(5 x 1 = 5)**
a)	State the chain rule for differentiation.	K1	CO1
b)	Define evolute of a curve.	K1	CO1
c)	State any two properties of definite integral.	K1	CO1
d)	State a result on Jacobians.	K1	CO1
e)	Define beta integral.	K1	CO1
2.	**Fill in the blanks**		**(5 x 1 = 5)**
a)	If u, v are functions of x, then $\frac{d}{dx}(uv) =$_____ .	K1	CO1
b)	The slope of the curve $r = e^{\theta}$ at $\theta = 0$ is _____ .	K1	CO1
c)	$\int e^{ax} sin(bx)\, dx =$_____ .	K1	CO1
d)	$\int_0^1 \int_0^1 dx\, dy =$_____ .	K1	CO1
e)	If n is a natural number, then $\Gamma(n+1) =$ _____.	K1	CO1
3.	**Choose the correct answer**		**(5 x 1 = 5)**
a)	What is $\frac{d}{dx}(\sin 2x)$? i. $\cos 2x$ ii. $\sin 2x$ iii. $\frac{\cos 2x}{2}$ iv. $\frac{\sin 2x}{2}$	K2	CO1
b)	What is the formula to find the subnormal of a curve at a point? i. $y\frac{dy}{dx}$ ii. $y \div \frac{dy}{dx}$ iii. $\frac{dy}{dx}$ iv. $\frac{dx}{dy}$	K2	CO1

c)	What is $\frac{\partial}{\partial x}(4x^2y + y^3)$? i. $2xy$ ii. $8xy$ iii. $4x^2 + 3y^2$ iv. 0	K2	CO1
d)	What is $\int_0^1 \int_0^1 \int_0^1 dx\, dy\, dz$? i. 0 ii. 1 iii. 2 iv. 3	K2	CO1
e)	What is $\beta(1,1)$? i. 0 ii. 6 iii. 1 iv. -1	K2	CO1
4.	**Say True or False.**	**(5 x 1 = 5)**	
a)	The n[th] derivative of e^{2x} is e^{2nx}.	K2	CO1
b)	The slope of the curve $y = mx + c$ is m.	K2	CO1
c)	The value of $\int_a^b f(x)\,dx$ is equal to $\int_a^b f(a + b - x)dx$.	K2	CO1
d)	The Jacobian matrix of a vector-valued function of several variables is the matrix of all its first-order partial derivatives.	K2	CO1
e)	The value of $\Gamma(\frac{1}{2}) = \sqrt{\pi}$.	K2	CO1
	SECTION B		
Answer any TWO of the following		**(2 x 10 = 20)**	
5.	Calculate the maximum and minimum values of the function $f(x,y) = 2(x^2 - y^2) - x^4 + y^4$.	K3	CO2
6.	Find the angle of intersection of the cardioids $r = a(1 + cos\theta)$ and $r = b(1 - cos\theta)$.	K3	CO2
7.	Show that $\int_0^{\frac{\pi}{2}} \frac{sin^{\frac{3}{2}}x}{sin^{\frac{3}{2}}x + cos^{\frac{3}{2}}x} dx = \frac{\pi}{4}$.	K3	CO2
8.	Prove that $\int_0^{\infty} e^{-x^2} dx = \frac{\sqrt{\pi}}{2}$.	K3	CO2
	SECTION C		
Answer any TWO of the following		**(2 x 10 = 20)**	
9.	Show that the maximum value of $x^2y^2z^2$ subject to the restriction $x^2+y^2 + z^2 = a^2$ is $\left(\frac{a^2}{3}\right)^3$.	K4	CO3

10.	Find the angle at which the radius vector cuts curve $\frac{l}{r} = 1 + e\cos\theta$.	K4	CO3
11.	Establish a reduction formula for $\int sin^m x\, cos^n x\, dx$, where m, n are positive integers.	K4	CO3
12.	Evaluate $\int_0^1 x^m (log(\frac{1}{x}))^n\, dx$.	K4	CO3
	SECTION D		
Answer any ONE of the following		**(1 x 20 = 20)**	
13.	Evaluate the minimum value of $u = a^3x^2 + b^3y^2 + c^3z^2$ where $\frac{1}{x} + \frac{1}{y} + \frac{1}{z} = 1$.	K5	CO4
14.	(a) By transforming into polar coordinates, evaluate $\int_0^\infty \int_0^\infty e^{-(x^2+y^2)}\, dxdy$. (10)	K5	CO4
	(b) Evaluate $\int\int\int \frac{dxdydz}{(x+y+z+1)^3}$ taken over the volume bounded by the planes $x = 0, y = 0, z = 0$ and $x + y + z = 1$. (10)	K5	CO4
	SECTION E		
Answer any ONE of the following		**(1 x 20 = 20)**	
15.	Show that in the parabola $y^2 = 4ax$ at the point *t*, the radius of curvature $\rho = -2a(1 + t^2)^{\frac{2}{3}}$ and the centre of curvature is $C(X, Y)$ where $X = 2a + 3at^3$ and $Y = -2at^3$. Hence deduce the equation of the evolute.	K6	CO5
16.	Deduce the value of $\beta(10,15)$ by establishing the relation between Beta and Gamma functions.	K6	CO5

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