LOYOLA COLLEGE (AUTONOMOUS), CHENNAI – 600 034

B.Sc. DEGREE EXAMINATION – **MATHEMATICS**

FOURTH SEMESTER – **APRIL 2022**

UMT 4501 – REAL ANALYSIS-I

Date: 16-06-2022 Dept. No. Max. : 100 Marks

Time: 09:00 AM - 12:00 NOON

PART – A

Answer ALL the Questions **(10 × 2 = 20)**

1. Let f: A → B and g : B → C be functions. Show that if g∘ f is injective then f is injective.
2. State principle of Mathematical Induction.
3. Define rational and irrational real numbers.
4. If a,b are real numbers show that $|ab| = |a||b|$.
5. Define supremum and infimum of a non empty subset of real numbers.
6. State completeness property of real numbers.
7. What is meant by Fibonacci sequence?
8. Give an example of a bounded sequence which is not convergent.
9. State Comparison Test for convergence of real sequences.
10. When do you say that a series of real numbers is conditionally convergent?

PART – B

Answer any FIVE Questions **(5 × 8 = 40)**

11. For each n ∈ N show that $\mathbf{1^2 + 2^2 + \cdots n^2 = \frac{n(n+1)(2n+1)}{6}}$.
12. Show that set of all rational numbers is denumerable.
13. Show that there does not exist a rational number r such that $\boldsymbol{r^2} = 2$.
14. Let a ≥ 0 and b ≥ 0, show that a < b ⇔ $\boldsymbol{a^2 < b^2 \Leftrightarrow \sqrt{a} < \sqrt{b}}$.
15. State and prove Archimedean Property.
16. State and prove nested intervals property.
17. Show that the series $\sum_{n=1}^{\infty} \frac{1}{n!}$ is convergent.
18. Let X = (x_n) be a sequence of real numbers and let m∈ $\boldsymbol{N}$. Show that the sequence X_m = (x_{m+n} : n∈N) converges if and only if X converges.

PART – C

Answer any TWO Questions **(2 × 20 = 40)**

19. (a) Prove that $n < 2^n$ for all n∈N.

(b) Show that the following statements are equivalent:

(i) S is a countable set

(ii) There exists a surjection of N into S.

(iii) There exists an injection of S into N. **(8+12)**

20. (a) Show that for positive real numbers a and b, $\sqrt{\boldsymbol{ab}} \le \frac{\boldsymbol{a+b}}{\boldsymbol{2}}$.

(b) If a and b are real numbers show that $\big||\boldsymbol{a}| - |\boldsymbol{b}|\big| \le |\boldsymbol{a} - \boldsymbol{b}|$. **(10+10)**

21 (a) If S is a subset of ℝ that contains at least two points and has the property if x,y ∈ S and x<y then [x , y] ⊆ $\boldsymbol{S}$, show that S is an interval.

(b) Show that [0,1] is not countable. **(10+10)**

22. (a) If a>0 , show that lim $(\frac{\boldsymbol{1}}{\boldsymbol{1+na}})$ = 0.

(b) Show that the series $\sum_{\boldsymbol{n=1}}^{\infty} \boldsymbol{a_n} \cos \boldsymbol{nx}$ converges if (a_n) is decreasing with lim (a_n) =0 and x ≠ 2kπ. **(10+10)**

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