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	Ş	FOURTH SEMESTER – APRIL 2022 UMT 4601 – COMBINATORICS							
LUCEAT LUX VESTRA	ß								
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Date: 23	3-06-2022	Dept	. No.					Max. : 100 Marks	
Time: 0	9:00 AM - 12:00	NOON					I		
				PAR	Г – А				
Answer ALL Questions:				$(10 \times 2 = 20)$					
1. Find <i>f</i> (<i>n</i> ,	1. Find $f(n, k)$ where $n = 4$ and $k = 2$. [using recurrence relation].								
2. How many	. How many different binary sequences of length 10 containing exactly 5 zeros?								
3. Tom has 7	5 books but enough	room on h	is book	shelf fo	or only 2	20. In ho	ow man	y ways can he fill his shelf?	
4. A binary se	I. A binary sequence of length <i>n</i> is a string of <i>n</i> digits each of which is 0 or 1. How many such sequences are there?								
5. Construct	2 different 5 \times 5 La	tin square	which	have th	e same	first rov	v		
6. 10 people	meet and form 5 pai	irs. How m	any wa	iys their	pairs c	an obtai	n?		
7. When a pa	ath or trail is said to b	e closed?							
8. Write any	one possible derange	ements of	1234	5.					
9. When a bo	pard is said to have a	forbidden	positio	n?					
10. Find the ro	ook polynomial of n–	non inters	ecting	2×2 b	locks.				
PART – B									
				Р	ART –	В			
Answer any	FIVF Questions			Р	ART –	В		$(5 \times 8 = 40)$	
Answer any	FIVE Questions.	and $(n -$	- <i>b</i> - 1	\mathbf{P}	ART -	B $(-1) t($	$n b \perp 1$	$(5 \times 8 = 40)$	
Answer any 11. Suppose t $t(n,k) = -\frac{1}{2}$	FIVE Questions. hat $t(n, n - 1) = 1$ $(n-1)^{n-k-1}(n-2)!$	and (<i>n</i> -	- <i>k</i> — 1	P _) t(n, k	ART -	B - 1) t(n, k + 1	(5 imes 8 = 40) L) for each $k < n-1$. Show that	
Answer any 11 . Suppose t $t(n,k) = -12$. If a footba	FIVE Questions. hat $t(n, n - 1) = 1$ $\frac{(n-1)^{n-k-1}(n-2)!}{(k-1)!(n-k-1)!}$. Il league of n teams.	and $(n -$	– <i>k –</i> 1 n plays	P .) <i>t</i> (<i>n</i> , <i>k</i> s each o	ART -)= $k(n)$	B -1 $t($	n, k+1	$(5 \times 8 = 40)$ 1) for each $k < n - 1$. Show that er of games played is therefore 2 <i>C</i> .	
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17. Find the general formula for U_n , the number of different rooted trees.

18. Given a chessboard C, choose any square of C and let D denote the board obtained by deleting from C every square in the same row or column at the chosen square (including the chosen square itself). Let E denote the board obtained from C by deleting only the chosen square. Then prove that R(x, C) = xR(x, D) + R(x, E).

PART – C

Answer any TWO Questions.

19. (a) Suppose that each of k-indistinguishable golf balls have to be colored with any one of n colours using binomial theorem (generating function) approach. Find out how many different colouring are possible and hence deduce the case k = 4 and n = 9. (12+8)

(b) Explain ordered selection and evaluate the following: (i) p(7,4), (ii) p(9,5)

20. (a) Let n be a positive integer. Show that if $(1+x)^n$ is expanded as a sum of powers on n, the coefficient of x^r is $\binom{n}{r}$

(b) Find a_n if $a_n = 4a_{n-1} + 4a_{n-2} - 16a_{n-3}$, $a_1 = 8$, $a_2 = 4$, $a_3 = 24$. (12+8)

21. (a) Find the value of
$$k_2$$
 given, $\left(\frac{\sqrt{5}+1}{2\sqrt{5}}\right)\left(\frac{1+\sqrt{5}}{2}\right) + k_2\left(\frac{1-\sqrt{5}}{2}\right) = 1.$

(b) State and prove Marriage Theorem.

22. (a) Find the rook polynomial of the board



(b) Derive $a_n = n! \left\{ 1 - \frac{1}{1!} + \frac{1}{2!} \right\}$ usion principle.

(10+10)

(a)a)a(a)a)a(a)a)a

$$1 - \dots + (-1)^n \frac{1}{n!}$$
 by using inclusion and exclusion

 $(2 \times 20 = 40)$

(8+12)