LOYOLA COLLEGE (AUTONOMOUS), CHENNAI – 600 034

B.Sc. DEGREE EXAMINATION – **MATHEMATICS**

FIFTH SEMESTER – **APRIL 2022**

UMT 5601 – GRAPH THEORY

Date: 21-06-2022 Dept. No. Max. : 100 Marks

Time: 09:00 AM - 12:00 NOON

Part – A **(10 x 2 = 20)**

Answer ALL Questions

1. Define Simple graph with an example.
2. When a graph is said to be complete?
3. Show that the maximum number of edges in a simple graph with n vertices is $\frac{n(n-1)}{2}$.
4. Define Hamiltonian path.
5. A graph with atleast one vertex is also called a tree. True or False. Justify
6. Define minimally connected graph.
7. Show that every cut set in a connected graph G must contain atleast one branch of every spanning tree T.
8. When do you say that a graph is separable?
9. Give two examples for Non-Jordan curve.
10. What is embedding in graph?

Part – B **(5 x 8 = 40)**

Answer any FIVE Questions

11. Prove that the maximum degree of any vertex in a graph G is with n vertices is $n-1$.
12. Define the following with two examples:
    a. Bipartite graph
    b. Complete Bipartite graph
    c. Maximum degree
    d. Minimum degree
13. Explain the operations union and intersection on graphs with two examples.
14. Write the following applications of graph theory with examples:
    a. Konigsberg bridge problem
    b. Utility problem
    c. Seating problem **(4+2+2)**
15. Prove that any connected graph with n vertices and $n-1$ edges is a tree.
16. Show that the vertex connectivity of a graph cannot exceed the edge connectivity of G.
17. In any simple, connected planar graph with f regions, n vertices and e edges $(e > 2)$, show that the following inequalities must hold: (i) $e \geq \frac{3}{2} f$; (ii) $e \leq 3n - 6$. **(4+4)**
18. Show that every bi-partite graph is $2-$chromatic.

Part – C **(2 x 20 = 40)**

Answer any TWO Question

19. (a) Show that a graph G is disconnected if and only if its vertex set V can be partitioned into two non-empty disjoint subsets V_1 and V_2 such that there exists no edge in G whose one end vertex is in V_1 and the other in subset V_2.

(b) Show that a simple graph with n vertices and k components can have at most $\frac{(n-k)(n-k+1)}{2}$ edges.

(10+10)

20. (a) Prove that a connected graph G is an Euler graph if and only if all vertices of G are of even degree.

(b) Show that the maximum vertex connectivity of a graph G with n vertices and e edges is the integral part of $\frac{2e}{n}$. **(10+10)**

21. (a) State and prove Euler's formula.

(b) Find the chromatic number and chromatic polynomial of trees with *n* vertices.

(10+10)

22. (a) Show that every tree with two or more vertices is $2-$chromatic.

(b) Show that the complete bipartite graph $K_{3,3}$ is non-planar. **(10+10)**

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