LOYOLA COLLEGE (AUTONOMOUS), CHENNAI – 600 034

B.Sc. DEGREE EXAMINATION – **MATHEMATICS**

SIXTH SEMESTER – **APRIL 2022**

UMT 6501 – COMPLEX ANALYSIS

Date: 15-06-2022 Dept. No. Max. : 100 Marks

Time: 01:00 PM - 04:00 PM

PART - A

Answer ALL Questions: **(10 X 2 = 20 Marks)**

1. Let $f(z) = iz/2$ in the open disc $|z| < 1$. Show that $\lim_{z\to 1} f(z) = \frac{i}{2}$
2. Verify the Cauchy – Riemann equations for the function $f(z) = e^z$.
3. Show that the function $f(z) = Re\ z$ is nowhere differentiable.
4. Let a function $f(z)$ be analytic in a domain D. Prove that if $\overline{f(z)}$ be analytic in D, then $f(z)$ must be a constant.
5. State Maximum Moduli Principle.
6. Define the convergence of sequences.
7. Find the singular points of the following function: $\frac{z+1}{z^3(z^2+1)}$.
8. State Cauchy's Residue theorem.
9. Define a linear fractional transformation.
10. When do you say that a function f is conformal at z_0 ?

PART - B

Answer Any FIVE Questions: **(5 X 8 = 40 Marks)**

11. Prove that Cauchy-Riemann equations are satisfied by a non-differentiable function $f(z) = \sqrt{|xy|}$ at $z = 0$.
12. Determine whether $f'(z)$ exists and find its value for $f(z) = \frac{1}{z}$ where $z = re^{i\theta}$.
13. Prove that $u = 2x - x^3 + 3xy^2$ is harmonic and find its harmonic conjugate. Also find the corresponding analytic function.
14. Let C be the arc of the circle $|z| = 2$ from $z = 2$ to $z = 2i$ that lies in the first quadrant. Prove that $\left|\int_C \frac{dz}{z^2+1}\right| \leq \frac{\pi}{3}$.
15. State Liouville's theorem and deduce the Fundamental theorem of algebra.
16. Expand the following function in a Laurent's series in $(i)\ 1 < |z| < 2$ and $(ii)\ |z| > 2$. $f(z) = \frac{z}{(z-1)(2-z)}$.
17. Find the residue of $\frac{e^z}{z^2(z^2+9)}$ at its poles.
18. Find the linear fractional transformation which maps $z_1 = 0,\ z_2 = -i$ and $z_3 = -1$ onto the points $w_1 = i,\ w_2 = 1$ and $w_3 = 0$.

PART – C

Answer Any TWO Questions: **(2 X 20 = 40 Marks)**

19. State and prove the necessary and sufficient condition for $f(z)$ to be differentiable at a point.

20. (a) Find the analytic function $f(z) = u + iv$ given that $u - v = e^z(\cos y - \sin y)$. (10 marks)

(b) Let C denote the boundary of the square whose sides lie along the lines $x = \pm 2$ and $y = \pm 2$ where C is described in the positive sense. Evaluate:

(i) $\int_C \frac{z\,dz}{2z+1}$ and (ii) $\int_C \frac{\cos z\,dz}{z(z^2+8)}$ (10 marks)

21. (a) State and prove Taylor's theorem. (10 marks)

(b) Using the method of Contour integration, evaluate:

$$\int_0^\infty \frac{\cos x}{(x^2+a^2)(x^2+b^2)}\,dx$$ (10 marks)

22. (a) State and prove Rouche's theorem. (10 marks)

(b) Show that the transformation $w = \frac{5-4z}{4z-2}$ maps the unit circle $|z| = 1$ onto a circle of radius unity and centre $-\frac{1}{2}$. (10 marks)

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