LOYOLA COLLEGE (AUTONOMOUS), CHENNAI – 600 034

B.Sc. DEGREE EXAMINATION – **MATHEMATICS**

SIXTH SEMESTER – APRIL 2022

UMT 6501 – COMPLEX ANALYSIS

Date: 15-06-2022 Dept. No. Time: 01:00 PM - 04:00 PM

PART - A

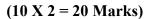
Answer ALL Questions:

- 1. Let f(z) = iz/2 in the open disc |z| < 1. Show that $\lim_{z \to 1} f(z) = \frac{i}{2}$
- 2. Verify the Cauchy Riemann equations for the function $f(z) = e^{z}$.
- 3. Show that the function f(z) = Re z is nowhere differentiable.
- 4. Let a function f(z) be analytic in a domain *D*. Prove that if $\overline{f(z)}$ be analytic in *D*, then f(z) must be a constant.
- 5. State Maximum Moduli Principle.
- 6. Define the convergence of sequences.
- 7. Find the singular points of the following function: $\frac{z+1}{z^{3}(z^{2}+1)}$
- 8. State Cauchy's Residue theorem.
- 9. Define a linear fractional transformation.
- 10. When do you say that a function f is conformal at z_0 ?

PART - B

Answer Any FIVE Questions:

- 11. Prove that Cauchy-Riemann equations are satisfied by a non-differentiable function $f(z) = \sqrt{|xy|}$ at z = 0.
- 12. Determine whether f'(z) exists and find its value for $f(z) = \frac{1}{z}$ where $z = re^{i\theta}$.
- 13. Prove that $u = 2x x^3 + 3xy^2$ is harmonic and find its harmonic conjugate. Also find the corresponding analytic function.
- 14. Let C be the arc of the circle |z| = 2 from z = 2 to z = 2i that lies in the first quadrant. Prove that $\left| \int_{C} \frac{dz}{z^{2}+1} \right| \leq \frac{\pi}{3}$.
- 15. State Liouville's theorem and deduce the Fundamental theorem of algebra.
- 16. Expand the following function in a Laurent's series in (i) 1 < |z| < 2 and (ii) |z| > 2. $f(z) = \frac{z}{(z-1)(2-z)}.$
- 17. Find the residue of $\frac{e^z}{z^2(z^2+9)}$ at its poles.
- 18. Find the linear fractional transformation which maps $z_1 = 0$, $z_2 = -i$ and $z_3 = -1$ onto the points $w_1 = i$, $w_2 = 1$ and $w_3 = 0$.



(5 X 8 = 40 Marks)

Max.: 100 Marks

PART – C

Answer Any TWO Questions:

19. State and prove the necessary and sufficient condition for f(z) to be differentiable at a point.

20. (a) Find the analytic function f(z) = u + iv given that $u - v = e^{z}(\cos y - \sin y)$. (10 marks)

(b) Let C denote the boundary of the square whose sides lie along the lines $x = \pm 2$ and $y = \pm 2$ where *C* is described in the positive sense. Evaluate:

(i)
$$\int_C \frac{z \, dz}{2z+1}$$
 and (ii) $\int_C \frac{\cos z \, dz}{z(z^2+8)}$ (10 marks)

21. (a) State and prove Taylor's theorem. (b) Using the method of Contour integration, evaluate:

$$\int_0^\infty \frac{\cos x}{(x^2 + a^2)(x^2 + b^2)} dx$$
 (10 marks)

22. (a) State and prove Rouche's theorem.

(b) Show that the transformation $w = \frac{5-4z}{4z-2}$ maps the unit circle |z| = 1 onto a circle of radius unity and centre $-\frac{1}{2}$. (10 marks)

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(2 X 20 = 40 Marks)

(10 marks)

(10 marks)