

**LOYOLA COLLEGE (AUTONOMOUS), CHENNAI – 600 034**



**B.Sc. DEGREE EXAMINATION – MATHEMATICS**

**SECOND SEMESTER – APRIL 2023**

**MT 2501 – ALGEBRA, ANAL.GEO. & CALCULUS - II**

Date: 10-05-2023

Dept. No.

Max. : 100 Marks

Time: 09:00 - 12:00 NOON

**SECTION A**

Answer **ALL** the questions:

**(10x2 = 20)**

1. Evaluate  $\int \log x \, dx$ .
2. Evaluate  $\int \frac{dx}{x^2+2x+5}$ .
3. Solve  $\sqrt{1+y^2} + \sqrt{1+x^2} \frac{dy}{dx} = 0$ .
4. Solve  $(D^2 - 5D + 6)y = 0$ .
5. Define convergent sequence with an example.
6. State Cauchy's root test.
7. Write down the expansion of  $(3x + 5y)^5$ .
8. Expand  $\log(1+x)$ .
9. Find the distance of the origin from the plane  $6x - 3y + 2z - 14 = 0$ .
10. Find the equation of the sphere with radius 4 and centre  $(1, 2, 3)$ .

**SECTION B**

Answer any **FIVE** questions:

**(5x8 = 40)**

11. Prove that  $\int_0^{\pi/4} \log(1 + \tan \theta) \, d\theta = \frac{\pi}{8} \log 2$ .
12. Evaluate  $\int \frac{x+\sin x}{1+\cos x} \, dx$ .
13. Solve  $\frac{dy}{dx} - y \tan x = \frac{\sin x \cos^2 x}{y^2}$ .
14. Solve  $(D^2 + 4)y = x \sin x$ .
15. Test the convergence of the series  $\sum_{n=0}^{\infty} \frac{n^3+1}{2^{n+1}}$ .
16. Sum the series  $1 + \frac{1+3}{2!} + \frac{1+3+3^2}{3!} + \frac{1+3+3^2+3^3}{4!} + \dots$
17. Find the equation of the plane passing through the points  $(3, 1, 2)$ ,  $(3, 4, 4)$  and perpendicular to the plane  $5x + y + 4z = 0$ .
18. Find the equation of the sphere which passes through the circle  $x^2 + y^2 + z^2 - 2x - 4y = 0$ ,  $x + 2y + 3z = 8$  and touches the plane  $4x + 3y = 25$ .

**SECTION C**

Answer any **TWO** questions:

**(2x20 = 40)**

19. (a) Find the reduction formula for  $I_n = \int \sin^n x \, dx$ , where  $n \in \mathbb{N}$  and hence find  $\int_0^{\pi/2} \sin^n x \, dx$ .  
(b) Evaluate  $I = \int_0^{\pi/2} \log \sin x \, dx$ .

**(10+10)**

20. (a) Solve  $(D^2 + 2D + 5)y = x e^x$ .

(b) Solve  $\frac{d^2y}{dx^2} + y = \sec x$ .

**(8+12)**

21. (a) Examine the convergence of  $\sum_{n=1}^{\infty} \left(\frac{n}{n+1}\right)^{\frac{1}{2}} x^n$ .

(b) Find the sum to the series  $\frac{5}{1!} + \frac{7}{3!} + \frac{9}{5!} + \dots$

**(10+10)**

22. (a) Prove that the lines  $\frac{x+1}{-3} = \frac{y+10}{8} = \frac{z-1}{2}$ ;  $\frac{x+3}{-4} = \frac{y+1}{7} = \frac{z-4}{1}$  are coplanar and find also their point of intersection and plane through them.

(b) Find the equation of the sphere through the four points (2, 3, 1), (5, -1, 2), (4, 3, 1) and (2, 5, 3).

**(10+10)**

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