LOYOLA COLLEGE (AUTONOMOUS), CHENNAI – 600 034

B.Sc. DEGREE EXAMINATION – **MATHEMATICS**

THIRD SEMESTER – **APRIL 2023**

MT 3501 – ALGEBRA, CALCULUS AND VECTOR ANALYSIS

Date: 02-05-2023 Dept. No. Max. : 100 Marks
Time: 01:00 PM - 04:00 PM

Section A

Answer ALL questions: **(10 × 2 = 20)**

1. Evaluate $\int_0^2 \int_1^x xydydx$.
2. Identify the value of $\int_0^{\pi/2} sin^7\theta cos^5\theta\, d\theta$.
3. Obtain a partial differential equation by eliminating *a*, *b* from *z* = (*x* + *a*)(*y* +*b*).
4. Solve $\frac{\partial z}{\partial x} = 0$.
5. Find $\nabla\varphi$, if $\varphi = xyz$.
6. State Stroke's theorem
7. Find the Laplace transform of *sint*.
8. Determine $L^{-1}\left[\frac{1}{s^2-9}\right]$.
9. Obtain the number of divisors of 360.
10. What is the remainder when 2^{1000} is divided by 17?

Section B

Answer any FIVE questions: **(5 × 8 = 40)**

11. Given that *x* + *y* = *u*, *y* = *uv*, change the variables to *u*, *v* in the integral $\iint (xy(1-x-y))^{1/2} dxdy$ taken over the area of the triangle with sides *x* = 0, *y* = 0, *x* + *y* = 1, and evaluate it.
12. Determine the value of $\iint (a^2 - x^2)\, dxdy$ over half the circle $x^2 + y^2 = a^2$ in the positive quadrant.
13. Solve $p^2 + q^2 = npq$.
14. Obtain the complete integral of the partial differential equation $pxy + pq + qy = yz$.
15. Use Green's theorem and evaluate $\int_C (xy + x^2)\, dx + (x^2 + y^2)dy$, where *C* is the square formed by the lines $x = -1, x = 1, y = -1, y = 1$ in the *xy*-plane.
16. Find the Laplace transform of $f(t) = \begin{cases} 0, & when\ 0 < t \leq 2 \\ 3, & when\ t > 2 \end{cases}$
17. Find the highest power of 3 dividing 1000! .
18. Show that if *n* is a prime number and *x* and *y* are both prime to *n*, then $x^{n-1} - y^{n-1}$ is divisible by *n*. Also, deduce that $x^{12} - y^{12}$ is divisible by 1365.

Section C

Answer any TWO questions: **($2 \times 20 = 40$)**

19. (a) Change the order of integration and find the value of $\int_0^a \int_{\frac{x^2}{a}}^{2a-x} xy\,dy\,dx$. (15 marks)

(b) Express $\int_0^1 x^m(1-x^n)^p dx$ in terms of Gamma functions. (5 marks)

20. (a) Solve the partial differential equation $p(1+q^2) = q(z-1)$. (8 marks)

(b) Find the general solution of $(y+z)p + (z+x)q = x+y$. (12 marks)

21. (a) If $\vec{v} = \vec{w} \times \vec{r}$ where $\vec{w}$ is a constant vector and $\vec{r} = x\vec{i} + y\vec{j} + z\vec{k}$, show that $\frac{1}{2} curl\vec{v} = \vec{w}$.

(5 marks)

(b) Verify Gauss-Divergence theorem for $\vec{F} = (x+y)\vec{i} + x\vec{j} + z\vec{k}$ taken over the region bounded by the planes $x = 0$, $x = 1$, $y = 0$, $y = 1$, $z = 0$, $z = 1$. (15 marks)

22. (a) Solve the equation $\frac{d^2y}{dt^2} + 2\frac{dy}{dt} - 3y = sint$ given that $y = \frac{dy}{dt} = 0$ when $t = 0$. (15 marks)

(b) State and prove Wilson's theorem. (5 marks)

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