

LOYOLA COLLEGE (AUTONOMOUS), CHENNAI – 600 034



M.Sc. DEGREE EXAMINATION – MATHEMATICS

THIRD SEMESTER – APRIL 2023

PMT 3601 – COMBINATORICS

Date: 08-05-2023

Dept. No.

Max. : 100 Marks

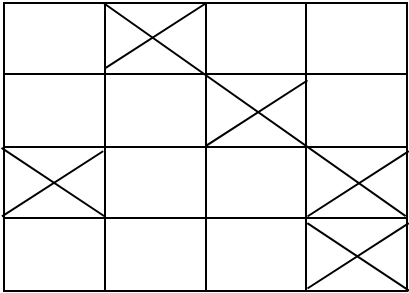
Time: 01:00 PM - 04:00 PM

PART – A

Answer all questions

(5 x 20 = 100 Marks)

1	(a)	Consider the letters of the word DELHI. How many words of length 1,2,3,4,5 can be formed whether or not meaningful, using the letters. a. repetition of letters allowed. b. repetition is not allowed.	(5)
(or)			
	(b)	How many partitions are possible for the number 10 and what are they?	(5)
	(c)	Derive the Stirling numbers of the first and second kind and tabulate the value for S_7^7	(15)
(or)			
	(d)	i.) In how many ways can 9 distinct objects be placed in 5 distinct boxes in such a way that 3 of these boxes would be occupied and 2 would be empty. ii.) How many integers between 1000 and 9999 inclusive have distinct digits? Of these, how many are even numbers? How many consists entirely of odd digits?	(15)
2	(a)	Verify that the number of increasing words of length 10 out of the alphabet {a, b, c, d, e} with $a < b < c < d < e$ is the coefficient of t^{10} in $(1 - t)^{-5}$. Try to explain why this is so.	(5)
(or)			
	(b)	If $\Phi_5(n)$ and $\Psi_5(n)$ denote the numbers of undirected and directed loopless graphs of order 5, with n edges, none of them a multiple edge, find the series $\sum \Phi_5(n)t^n$ and $\sum \Psi_5(n)t^n$.	(5)
	(c)	Briefly explain the difference between ordinary generating function and the exponential generating function with an example.	(15)
(or)			
	(d)	In how many ways can a total of 16 be obtained by rolling 4 dice once?	(15)
3	(a)	Find the coefficient of $\alpha_1^2 \alpha_2^2 \alpha_3^2 \alpha_4$ in $h_4 h_3$ by computing $\phi_{43,2221}$.	(5)
(or)			

	(b) Obtain determinantal expression for s_r in terms of a. the a_r 's and b. the h_r 's	(5)
	(c) Briefly explain the four types of symmetric functions.	(15)
	(or)	
	(d) Given $\lambda \vdash N$ prove that k_λ is a linear combination of the s_μ 's.	(15)
4	(a) Five gentle man A, B, C, D, E attend a party, where, before joining the party they leave their overcoats in a checkroom. After the party, the overcoats get mixed up and are returned to the gentlemen in a random manner. What is the probability that none receives his own overcoat?	(5)
	(or)	
	(b) Find the number of positive integers, not greater than 100, which are not divisible by 2, 3, or 5.	(5)
	(c) State and prove generalized inclusion and exclusion principle	(15)
	(or)	
	(d) If a chess board has 4×4 boxes in it with following forbidden positions  Find the rook's polynomial.	(15)
5	(a) How many distinct circular necklace patterns are possible with 6 beads, these beads being available in three different colors? Can you make an inventory of them?	(5)
	(or)	
	(b) Find the cycle structure of the permutations on the set of 4 vertices, 4 faces and 6 edges of the regular tetrahedron induced by 12 rotations – (besides identity, eight of them each through 120° around an axis joining a vertex to the centroid of the opposite face, and three of them each through 180° around an axis joining the middle points of the opposite edges).	(5)
	(c) State and prove Burnside's lemma.	(15)
	(or)	
	(d) State and prove Polya's enumeration theorem.	(15)

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