LOYOLA COLLEGE (AUTONOMOUS), CHENNAI – 600 034

M.Sc. DEGREE EXAMINATION – **MATHEMATICS**

SECOND SEMESTER – **APRIL 2023**

PMT2ME02 – PARALLEL INTERCONNECTION NETWORKS

Date: 08-05-2023 Dept. No. Max. : 100 Marks
Time: 01:00 PM - 04:00 PM

SECTION A – K1 (CO1)

Answer ALL the questions **(5 x 1 = 5)**

1. **Answer the following**

a) Define interconnection network.

b) What is subdivision of an edge?

c) Define weight of a vertex x in a hypercube Q_n.

d) Define Benes network.

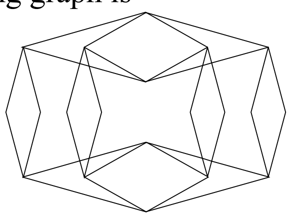
e) Define routing of a graph G.

SECTION A – K2 (CO1)

Answer ALL the questions **(5 x 1 = 5)**

2. **Choose the correct answer**

a) The dominating number of the following graph is



(a) 3 (b) 4 (c) 5 (d) 6

b) The complete graph K_n and complete bipartite graph $K_{m,n}$ are
(a) vertex-transitive (b) edge-transitive
(c) both (a) and (b) (d) neither (a) nor (b)

c) The de Bruijn network of diameter 8 and degree 8 can interconnect ………… processors
(a) 565 (b) 5665 (c) 656 (d) 65556

d) The diameter of CCC(n) is
(a) $\left\lfloor \frac{1}{2}(5n-2) \right\rfloor$ (b) $\left\lfloor \frac{1}{2}(5n+2) \right\rfloor$ (c) $\lfloor (5n+1) \rfloor$ (d) $\lfloor (5n-1) \rfloor$

e) The forwarding index of a star $K_{1,n-1}$, is
(a) $(n-1)(n-2)$ (b) $n\,(n-1)$
(c) $(n+1)\,n$ (d) $(n+1)(n+2)$

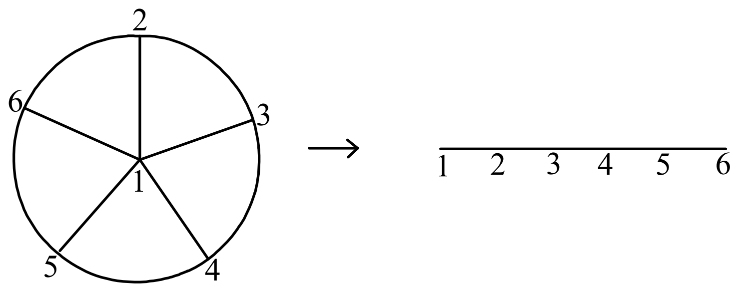
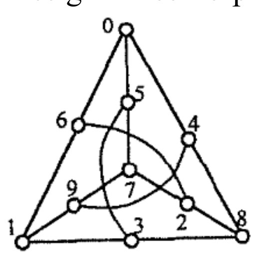
SECTION B – K3 (CO2)

Answer any THREE of the following **(3 x 10 = 30)**

3. a) Let X and Y be subsets of V(G). Then prove that $d_G^+(X) = d_G^-(X)$ if G is a balanced diagraph.
b) Let G be a strongly connected digraph with order $n(\geq 2)$ and the maximum degree. Then prove that $d(G) = \begin{cases} = n-1 & for\ d = 1 \\ \geq \lceil log_d(n(d-1)+1) \rceil - 1 & for\ d \geq 2 \end{cases}$ (5 + 5)

4. Prove that the converse of $\overleftarrow{C_\Gamma(S)}$ of a cayley graph $C_\Gamma(S)$ is also a cayley graph. Also list 5 properties of a cayley graph.

5. Define the de Bruijn Network B (d, n). Find the number of vertices and edges in B (2, n). Sketch B (2, 3).

6.	a) Define a mesh, cylinder and Torus networks of dimension $m \times n$. Also, draw a mesh, cylinder and torus of dimension 4 x 4. b) Draw the 3-dimesnional Benes network BB (3). (5 + 5)
7.	a) Write a note on surviving route graph and give an example. b) Find the forwarding index of the directed cycle C_n. (5 + 5)
	SECTION C – K4 (CO3)
	Answer any TWO of the following **(2 x 12.5 = 25)**
8.	Define (i) dilation of an embedding and (ii) congestion of an embedding. For the embedding *f* of a wheel on 6 vertices onto a path on 6 vertices, find the dilation, congestion, dilation-sum and congestion-sum. 
9.	Give an example of an edge-transitive graph which is not vertex-transitive. Prove that every edge-transitive graph is either vertex - transitive or bipartite.
10.	Let T_n be a binary tree of height n, $n \geq 2$ prove that i. T_n cannot be embedded into Q_{n+1} with dilation 1 ii. $2T_{n-1}$ can be embedded into Q_{n+1} with dilation 1 iii. T_n can be embedded into Q_{n+1} with dilation 2
11.	Define a Butterfly network (BF(n)) of dimension n. Find the number of vertices and edges in BF(n). Is BF(n) eulerian? Justify. Draw the diamond structure of BF(4).
	SECTION D – K5 (CO4)
	Answer any ONE of the following **(1 x 15 = 15)**
12.	Define the n-dimensional cube-connected cycle CCC(n). Find the number of vertices and edges in CCC(n). Is CCC(n) eulerian? Justify. Draw CCC(3). Draw wrapped Butterfly WBF(3).
13.	Let G be a strongly connected digraph with order n, prove that $\frac{1}{n}\sum_{y \in V}\sum_{x(\neq y)\in V}(d(G;x,y)-1) \leq \tau(G) \leq (n-1)(n-2)$. Also prove that the upper bound can be attained and, the lower bound of $\tau(G)$ can be attained if and only if there exists a minimum routing ρ_m in G for which the load of all vertices is the same.
	SECTION E – K6 (CO5)
	Answer any ONE of the following **(1 x 20 = 20)**
14.	a. If G is a connected undirected graph of order n and minimum degree δ, then prove that $d(G) \leq \frac{3n}{\delta+1}$. b. Design an isomorphic graph for the following graph having crossing number as 5. 

	c. Generate the Cayley graph when G = {1, -1, i, -i} under multiplication and S = {-1, i}. (5 + 5 + 10)
15.	a. Define a hypercube Q_n using binary sequence and cartesian product. Prove that the two definitions are equivalent. Draw Q_4 and also propose a shortest path between 0100110 and 1111001 in hypercube Q_7. Is this path unique? Justify. b. For any given vertex x of Q_n, prove that there exists a unique vertex y such that the distance $d(Q_n; x, y) = n$. Also prove that there is n internally disjoint (x, y)- paths of length n. (10 + 10)

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