LOYOLA COLLEGE (AUTONOMOUS), CHENNAI – 600 034

B.Sc. DEGREE EXAMINATION – **MATHEMATICS**

THIRD SEMESTER – **APRIL 2023**

UMT 3502 – DIFFERENTIAL EQUATIONS AND LAPLACE TRANSFORM

Date: 04-05-2023 Dept. No. Max. : 100 Marks
Time: 01:00 PM - 04:00 PM

SECTION A			
Answer ALL the Questions			
1.	**Answer the following** **(5 x 1 = 5 Marks)**		
a)	Give the solution of the equation $\frac{dy}{dx}=\frac{1-y}{1+x}$.	K1	CO1
b)	Find the solution of $(D^2+D+1)y=0$.	K1	CO1
c)	Write down the rule for integrating $Pdx+Qdy+Rdz$.	K1	CO1
d)	Write the Laplace transform integral for periodic functions.	K1	CO1
e)	Find $L^{-1}\left(\frac{s}{s^2+b^2}\right)$.	K1	CO1
2.	**Fill in the blanks** **(5 x 1 = 5 Marks)**		
a)	The solution of the differential equation $(xdy+ydx)=0$ is ____________.	K1	CO1
b)	The Complementary Function of the roots of the differential equation 2,2 and 3 is ____________.	K1	CO1
c)	The solution of the partial differential equation $\frac{\partial^2 z}{\partial y^2}=\sin y$ is ____________.	K1	CO1
d)	$L^{-1}\left(\frac{s}{s^2+k^2}\right)$ is ____________.	K1	CO1
e)	The $L\left\{\frac{t^2}{5}\right\}$ ____________.	K1	CO1
3.	**Choose the correct answer for the following** **(5 x 1 = 5 Marks)**		
a)	The degree of the differential equation $\left(\frac{d^3y}{dx^3}\right)^{5/2}+\left(\frac{d^2y}{dx^2}\right)^6+y=0$ is (i) 5 (ii) 2 (iii) 3 (iv) 6	K2	CO1
b)	The particular integral of the differential equation is $(D^2-4)y=e^{2x}$ is (i) $\frac{x}{4}$ (ii) $\frac{x}{2}$ (iii) $\frac{x^2}{2}$ (iv) $\frac{x^2}{4}$	K2	CO1
c)	The solution of the partial differential equation $\frac{\partial^2 z}{\partial y^2}=-\sin y$ is (i) $z=siny+\varphi(x)$ (ii) $z=-siny+\varphi(x)$ (iii) $z=siny+yf(x)+\varphi(x)$ (iv) $z=-siny+yf(x)+\varphi(x)$	K2	CO1
d)	$L\{\cosh 2t\}$ is (i) $\frac{s}{s^2+4}$ (ii) $\frac{2}{s^2+4}$ (iii) $\frac{s}{s^2-4}$ (iv) $\frac{2}{s^2-4}$	K2	CO1
e)	$L^{-1}\left\{\frac{1}{(s+a)^2}\right\}$ is (i) te^{-at} (ii) te^{at} (iii) t^2e^{-at} (iv) t^2e^{at}	K2	CO1
4.	**Say TRUE or FALSE** **(5 x 1 = 5 Marks)**		
a)	The differential equation of force F where m is mass and v velocity is given by	K2	CO1

	$F = \frac{d}{dt}(mv)$.		
b)	The solution of the equation $r\frac{dp}{dr} + 2p = 0\ is\ 2c$.	K2	CO1
c)	The complementary function of the differential equation $(D^2+4)y = 0$ has imaginary roots.	K2	CO1
d)	$L\{t^3\} = \frac{3}{s^{n+1}}$	K2	CO1
e)	Inverse Laplace transform is used to solve differential equations.	K2	CO1

SECTION B

Answer any TWO of the following: (2 x 10 = 20 Marks)

5.	Solve the equation $\sqrt{1+x^2}\,dx + \sqrt{1+y^2}\,dy = 0$.	K3	CO2
6.	Solve: $(D^2+4D+5)y = e^x + x^3 + \cos 2x$.	K3	CO2
7.	Solve: $x^2\frac{d^2y}{dx^2} + 4x\frac{dy}{dx} + 2y = e^x$.	K3	CO2
8.	Solve: $pxy + pq + qy = yz$.	K3	CO2

SECTION C

Answer any TWO of the following: (2 x 10 = 20 Marks)

9.	Solve: $z = px + qy + \sqrt{1+p^2+q^2}$.	K4	CO3
10.	Solve: $p\cot x + q\cot y = \cot z$.	K4	CO3
11.	Find the transform of rectangular wave given by $f(t) = \begin{cases} 1 & 0 < t < b \\ -1 & b < t < 2b \end{cases}$	K4	CO3
12.	Find (i) $L\{t^2cos^2t\}$ (ii) $L\{\sin at - at\cos at\}$	K4	CO3

SECTION D

Answer any ONE of the following: (1 x 20 = 20 Marks)

13.	a)	A particle falls under gravity in a resisting medium whose resistance varies with velocity. Find the relation between distance and velocity if initially the particle starts from rest.	K5	CO4
	b)	Evaluate the differential equation to find the solution of y in $x\frac{dy}{dx} + y\,logx = e^x x^{1-1/2\log x}$. (10+10)	K5	CO4
14.	a)	Applying the variation of parameters find the solution of $\frac{d^2y}{dx^2} + 4y = \tan 2x$.	K5	CO4
	b)	Solve the equation $(x-1)\frac{d^2y}{dx^2} - x\frac{dy}{dx} + y = (x-1)^2$, given that x and e^x are the integrals of the equation without the right-hand member. (10+10)	K5	CO4

SECTION E

Answer any ONE of the following (1 x 20 = 20 Marks)

15.	a)	Applying Lagrange's equation solve, $(x^2-yz)p + (y^2-zx)q = z^2-xy$.	K6	CO5
	b)	Formulate the Charpit's method to solve, $p^2+q^2-2px-2qy+1=0$. (10+10)	K6	CO5
16.		A particle moving a xy plane such that the position (x,y) at any point is given by $\frac{d^2y}{dt^2} + 4\frac{dy}{dt} - 5y = 5$, with initial displacement is zero and initial velocity is 2. Determine the value of y satisfying the equation.	K6	CO5

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