LOYOLA COLLEGE (AUTONOMOUS), CHENNAI – 600 034

B.Sc. DEGREE EXAMINATION – **STATISTICS**

FOURTH SEMESTER – **APRIL 2023**

UMT 4403 – MATHEMATICS FOR STATISTICS - II

Date: 04-05-2023 Dept. No. Max. : 100 Marks

Time: 09:00 AM - 12:00 NOON

PART – A

Answer all the questions: **$(10 \times 2 = 20)$**

1. Define limit of a sequence.
2. Define bounded sequence.
3. What is absolute convergence of a series?
4. What is conditional convergence of a series?
5. When do we say that a function is monotone?
6. When do we say that a function is strictly increasing?
7. Is every differentiable function continuous? – Justify.
8. What is a derivative of a function at a point?
9. What is an upper sum of a function?
10. Define measure zero.

PART B

Answer any Five of the following: **$(5 \times 8 = 40)$**

11. a.) If $\{s_n\}_{n=0}^{\infty}$ is a sequence of non-negative numbers and $\lim_{n\to\infty} s_n = L$, then prove that $L \geq 0$. **(7 Marks)**

    b.) Determine the limit of the sequence $\left\{\frac{1}{(n+1)^2}\right\}_{n=0}^{\infty}$. **(3 Marks)**

12. If $\{s_n\}_{n=0}^{\infty}$ is a sequence of real numbers which converges to L, then show that $\{s_n^2\}_{n=1}^{\infty}$ converges to L^2.
13. Prove that the series $\sum_{n=1}^{\infty} x^n$ converges to $\frac{1}{1-x}$ if $0 < x < 1$ and diverges if $x \geq 1$.
14. If $\sum_{n=1}^{\infty} a_n$ converges to A and $\sum_{n=1}^{\infty} b_n$ converges to B then show that $\sum_{n=1}^{\infty} a_n + b_n$ converges to $A + B$ and $\sum_{n=1}^{\infty} ca_n$ converges to cA where $c \in \mathbb{R}$.
15. If f is a non-decreasing function on the bounded open interval (a, b) and f is bounded above on (a, b), then show that $\lim_{x\to b^-} f(x)$ exist.
16. If f and g both have derivatives at $c \in \mathbb{R}$,then show that $(fg)'(c) = f'(c)g(c) + f(c)g'(c)$ and $(f + g)'c = f'(c) + g'(c)$.
17. State and Prove Taylor's Formula.
18. If $f \in \mathbb{R}[a, b]$, $g \in \mathbb{R}[a, b]$, then prove that $f + g \in \mathbb{R}[a, b]$ and $\int_a^b f + g = \int_a^b f + \int_a^b g$.

PART – C

Answer any Two of the following: $(\mathbf{2 \times 20 = 40})$

19. If $\{s_n\}_{n=1}^{\infty}$ and $\{t_n\}_{n=1}^{\infty}$ are sequence of real numbers where $\lim_{n\to\infty} s_n = L$ and $\lim_{n\to\infty} t_n = M$ and $c \in \mathbb{R}$ then prove the following
   a) $\lim_{n\to\infty} s_n + t_n = L + M.$
   b) $\lim_{n\to\infty} cs_n = cL.$

20. Prove that if $\sum_{n=1}^{\infty} a_n$ be a series of nonzero real numbers. Let $a = \lim_{n\to\infty} \inf\left|\frac{a_{n+1}}{a_n}\right|$, and $A = \lim_{n\to\infty} \sup\left|\frac{a_{n+1}}{a_n}\right|$ then
   a) If $A < 1$, then $\sum_{n=1}^{\infty} |a_n| < \infty$
   b) If $a > 1$ then $\sum_{n=1}^{\infty} a_n$ diverges
   c) If $a \le 1 \le A$, then the test fails.

21. State and prove Rolle's Theorem and use it to find a point c for the function $f(x) = (x-a)(b-x)$ such that $f'(c) = 0$.

22. a) If $f \in \mathbb{R}[a,b]$ and λ is any real number, then prove that $\lambda f \in \mathbb{R}[a,b]$ and $\int_a^b \lambda f = \int_a^b f$. **(15 Marks)**

    b) Evaluate $\int_0^1 (2x^2 - 3x + 5)dx$. **(5 Marks)**

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