LOYOLA COLLEGE (AUTONOMOUS), CHENNAI – 600 034

B.Sc. DEGREE EXAMINATION – **MATHEMATICS**

FOURTH SEMESTER – **APRIL 2023**

UMT 4501 – REAL ANALYSIS-I

Date: 02-05-2023 Dept. No. Max. : 100 Marks

Time: 09:00 AM - 12:00 NOON

PART A

Answer ALL the questions **(10 x 2 =20)**

1. Let $f: R \to R$ be defined by $f(x) = 2x - 3$ and $g: R \to R$ be defined by $g(x) = \frac{x+3}{2}$. Verify $f \circ g = g \circ f$.
2. State well ordering property of N.
3. Determine the set of all real numbers x such that $2x + 3 \leq 6$.
4. If z and a are elements in $\mathbb{R}$ with $z + a = a$ then show that $z = 0$.
5. Define supremum of a set $S \subseteq R$.
6. State completeness property of R.
7. Show that $\lim_{n\to\infty} \left(\frac{3n+2}{n+1}\right) = 3$.
8. Write the first five terms of the sequence $(x_n) = \frac{1}{(n+2)(3n+1)}$.
9. State Ratio test.
10. When a series is said to be absolutely convergent?

PART B

Answer any FIVE questions **(5 x 8 =40)**

11. Prove that $1^2 + 2^2 + \cdots + n^2 = \frac{n(n+1)(2n+1)}{6}$ is true for all $n \in N$ using principle of mathematical induction.
12. Prove that the set of all rational numbers is denumerable.
13. State and prove Bernoulli's inequality.
14. If $a, b \in R$ then prove that (i) $\left||a| - |b|\right| \leq |a - b|$ (ii) $|a - b| \leq |a| + |b|$.
15. Prove that a number u is the supremum of a nonempty subset S of $\mathbb{R}$ if and only if u satisfies the condition (i) $s \leq u$ for all $s \in S$ (ii) if $v < u$ then there exists $s' \in S$ such that $v < s'$.
16. State and prove Archimedean property.
17. State and prove Squeeze theorem.
18. Test for convergency of the series $\sum_{n=1}^{\infty} \frac{2 \cdot 4 \cdot 6 \cdot \ldots \cdot 2n}{1 \cdot 3 \cdot 5 \ldots \cdot (2n+1)}$ using Raabe's test.

PART C

Answer any TWO questions **(2 x 20 =40)**

19. (a) Prove that the following statements are equivalent

(i) S is a countable set

(ii) There exists a surjection of N onto S.

(iii) There exists an injection of S into N.

(b) State and prove Cantor's theorem. (12+8)

20. (a) If AM and GM denote the arithmetic and geometric mean of two positive real numbers a and b respectively, then prove that (i) $\frac{1}{2}(a+b) \geq \sqrt{ab}$ (ii) $\frac{1}{2}(a+b) = \sqrt{ab}$ if and only if $a = b$.

(b) State and prove Nested interval property. (10+10)

21. (a) Prove that the set of all real numbers R is not countable.

(b) Let $X = \{x_n\}$ and $Y = \{y_n\}$ be sequences of real numbers that converges to x and y respectively and let $c \in R$ then prove that the sequences $X + Y$, XY and cX converges to $x + y$, xy and cx respectively.

(8+12)

22. (a) State and prove Bolzano-Weierstrass theorem.

(b) Test the convergence for the series $\sum_{n=1}^{\infty} \frac{1}{n(n+1)}$ by Cauchy's integral test. (10+10)

$$$$$$$